codifies the empirical information concerning the rod, is first obtained from the identity
\[ g_{ik}' = g_{ik} - F_{ik}. \]

(ii) Reichenbach's Theory of Equivalent Descriptions.

A metric geometry in conjunction with a specified congruence definition given by a statement about the \( F_{ik} \) describes the coincidence behavior of a transported rod. Since the same facts of coincidence can be described in a linguistically alternative way by a different geometry when coupled with a suitably different congruence definition, Reichenbach speaks of different geometric descriptions which have the same factual content as "logically equivalent" [69, pp. 374–375] or simply as "equivalent" [73, pp. 138ff]. More generally, since not only spatial and temporal congruence but also metrical simultaneity are conventional in the sense of Section 2, part (i) above, geochronometries based on different definitions of congruence and/or simultaneity are equivalent. Among the equivalent descriptions of space, Reichenbach calls the one employing the customary definition of congruence the "normal system" and the particular geometry appropriate to it the "natural geometry" [73, p. 134]. The choice of a particular member from within a class of equivalent descriptions is a matter of convention and hence decided on the basis of convenience. But, as Reichenbach points out correctly, the decision as to which class among nonequivalent classes of equivalent descriptions is true is not at all conventional but is a matter of empirical fact. Thus, a Euclidean description and a certain non-Euclidean one cannot both be the natural geometry of a given space simultaneously: if a normal system is employed, they cannot both be true but will be the respective "normal" representatives of two nonequivalent classes of equivalent descriptions. On the other hand, all of the members of a particular class of equivalent descriptions obviously have the same truth value.

Reichenbach’s characterization of the empirical status of various geochronometries in terms of equivalence classes of descriptions is seen to be correct in the light of our preceding analysis, and it is summarized here because of its usefulness in the further pursuit of our inquiry.\(^{25}\)

\(^{25}\) Our endorsement of Reichenbach’s theory of equivalent descriptions in the context of geochronometry should not be construed as an espousal of his application of it to the theory of the states of macro-objects like trees during times when no human being observes them [70, pp. 17–20]. Since a critical discussion of Reichenbach’s account of unobserved macro-objects would carry us too far afield in this essay, suffice it to say the following here. In my judgment, Reichenbach’s application of the theory of equivalent descriptions to unobserved objects lacks clarity on precisely those points on which its relevance turns. But as I interpret the doctrine, I regard it as fundamentally incorrect because its ontology is that of Berkeley’s esse est percipi.
of $\sec \theta$ which exceed 1 each impart a *Euclidean* geometry to the table top no less than does the standard congruence given by

$$ds^2 = dx^2 + dy^2.$$ 

Accordingly, our demonstration will show that the requirement of Euclideanism does *not* uniquely determine a congruence class of intervals but allows an *infinity* of incompatible congruences. We shall therefore have established that there are infinitely many ways in which a measuring rod could squirm under transport on the table top as compared to its familiar *de facto* behavior while still yielding a Euclidean geometry for that surface.

To carry out the required demonstration, we first note the preliminary fact that the geometry yielded by a particular metrization is clearly independent of the particular coordinates in which that metrization is expressed. And hence if we expressed the standard metric

$$ds^2 = dx^2 + dy^2$$

in terms of the primed coordinates $x'$ and $y'$ given by the transformations

$$x = x' \sec \theta$$
$$y = y',$$

obtaining

$$ds^2 = \sec^2 \theta \, dx'^2 + dy'^2,$$

we would obtain a *Euclidean* geometry as before, since the latter equation would merely express the original standard metric in terms of the primed coordinates. Thus, when the same *invariant* $ds$ of the standard metric is expressed in terms of both primed and unprimed coordinates, the metric coefficients $g_{ik}'$ given by $\sec^2 \theta$, 0 and 1 yield a Euclidean geometry no less than do the unprimed coefficients 1, 0 and 1.

This elementary ancillary conclusion now enables us to see that the following *non*-standard metrization (or remetrization) of the surface in terms of the *original*, unprimed rectangular coordinates must likewise give rise to a Euclidean geometry:

$$ds^2 = \sec^2 \theta \, dx^2 + dy^2.$$ 

For the value of the Gaussian curvature and hence the prevailing geometry depends *not* on the particular coordinates (primed or unprimed) to which the metric coefficients $g_{ik}$ pertain but only on the *functional form* of the $g_{ik}$ [42, p. 281], which is the same here as in the case of the $g_{ik}'$ above.

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More generally, therefore, the geometry resulting from the *standard* metrization is also furnished by the following kind of *non*-standard metrization of a space of points, expressed in terms of the same (unprimed) coordinates as the standard one: the *non*-standard metrization has unprimed metric coefficients $g_{ik}$ which have the same *functional form* (to within an arbitrary constant arising from the choice of unit of length) as those primed coefficients $g_{ik}'$ which are obtained by expressing the standard metric in some set or other of *primed* coordinates via a suitable coordinate transformation. In view of the large variety of allowable coordinate transformations, it follows at once that the class of *non*-standard metrizations yielding a Euclidean geometry for a table top is far wider than the already infinite class given by

$$ds^2 = \sec^2 \theta \, dx^2 + dy^2,$$

where $\sec^2 \theta > 1$.

Thus, for example, there is identity of *functional form* between the standard metric in *polar* coordinates, which is given by

$$ds^2 = d\rho^2 + \rho^2 d\theta^2,$$

and the *non*-standard metric in *Cartesian* coordinates given by

$$ds^2 = dx^2 + x^2 dy^2,$$

since $x$ plays the same role formally as $\rho$, and similarly for $y$ and $\theta$. Consequently, the latter non-standard metric issues in a *Euclidean* geometry just as the former standard one does.

It is clear that the multiplicity of metrizations which we have proven for Euclidean geometry obtains as well for each of the non-Euclidean geometries. The failure of a geometry of two or more dimensions to determine a congruence definition uniquely does *not*, however, have a counterpart in the one-dimensional time continuum: the demand that Newton's laws hold in their customary metrical form determines a unique definition of temporal congruence. And hence it is feasible to rely on the law of translational or rotational inertia to define a time metric or "uniform time."

On the basis of this result, we can now show that a number of claims made by Reichenbach and Carnap respectively are false.

1. In 1951, Reichenbach wrote: "If we change the coordinative definition of congruence, a different geometry will result. This fact is called the *relativity of geometry*" [73, p. 132]. That this statement is false is evident from the fact that if, in our example of the table top, we change our con-
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guence definition from \( ds^2 = dx^2 + dy^2 \) to any one of the infinitely many definitions incompatible with it that are given by \( ds^2 = \sec^2 \theta dx^2 + dy^2 \), precisely the same Euclidean geometry results. Thus, contrary to Reichenbach, the introduction of a nonvanishing universal force corresponding to an alternative congruence does not guarantee a change in the geometry. Instead, the correct formulation of the relativity of geometry is that in the form of the \( ds \) function the congruence definition uniquely determines the geometry, though not conversely, and that any one of the congruence definitions issuing in a geometry \( G' \) can always be replaced by infinitely many suitably different congruences yielding a specified different geometry \( G \). In view of the unique fixation of the geometry by the congruence definition in the context of the facts of coincidence, the repudiation of a given geometry in favor of a different one does indeed require a change in the definition of congruence. And the new congruence definition which is expected to furnish the new required geometry can do so in one of the following two ways: (i) by determining a system of geodesics different from the one yielded by the original congruence definition, or (ii) if the geodesics determined by the new congruence definition are the same as those associated with the original definition, then the angles congruences must be different, i.e., the new congruence definition will have to require a different congruence class of angles. (For the specification of the magnitudes assigned to angles by the components \( g_{ik} \) of the metric tensor, see [27, pp. 37–38].)

That (ii) constitutes a genuine possibility for obtaining a different geometry will be evident from the following example in which two incompatible definitions of congruence

\[
\begin{align*}
&ds_1^2 = g_{ik} dx^i dx^k, \\
&ds_2^2 = g_{ik} dx^i dx^k,
\end{align*}
\]

yield the same system of geodesics via the equations \( \delta f ds_1 = 0 \) and \( \delta f ds_2 = 0 \) and yet determine different geometries (Gaussian curvatures) because they require incompatible congruence classes of angles appropriate to these respective geometries. A horizontal surface which is a Euclidean plane on the customary metrization can alternatively be metrized to have the geometry of a hemisphere by projection from the center of a sphere through the lower half of the sphere whose south pole is resting on that plane. Upon calling congruent on the horizontal surface segments and angles which are the projections of equal segments and angles respec-

tively on the lower hemisphere, the great circle arcs of the hemisphere map into the Euclidean straight lines of the plane such that every straight of the Euclidean description is also a straight (geodesic) of the new hemispherical geometry conferred on the horizontal surface. But the angles which are regarded as congruent on the horizontal surface in the new metrization are not congruent in the original metrization yielding a Euclidean description.

It must be pointed out, however, that if a change in the congruence definition preserves the geodesics, then its issuance in a different congruence class of angles is only a necessary and not a sufficient condition for imparting to the surface a metric geometry different from the one yielded by the original congruence definition. This fact becomes evident by reference to our earlier case of the table top's being a model of Euclidean geometry both on the customary metric \( ds^2 = dx^2 + dy^2 \) and on the different metric \( ds^2 = \sec^2 \theta dx^2 + dy^2 \): the geodesics as well as the geometries furnished by these incompatible metrics are the same, but the angles which are congruent in the new metric are generally not congruent in the original one. That these two metrics issue in incompatible congruence classes of angles though in the same geometry can be seen as follows: a Euclidean triangle which is equilateral on the new metric \( ds' \) will not be equilateral on the customary one \( ds \), and hence the three angles of such a triangle will all be congruent to each other in the former metric but not in the latter.

It is clear now that an arbitrary change in the congruence definition for either line segments or angles or both cannot as such guarantee a different geometry.

2. Reichenbach explicitly asserts incorrectly that the geometry uniquely determines a congruence definition appropriate to it. Says he: "There is nothing wrong with a coordinative definition established on the requirement that a certain kind of geometry is to result from the measurements. . . . A coordinative definition can also be introduced by the prescription what the result of the measurements is to be. "The comparison of length is to be performed in such a way that Euclidean geometry will be the result'—this stipulation is a possible form of a coordinative definition" [72, pp. 33–34]. And in reply to Hugo Dingler's contention [18, p. 50] that the rigid body is uniquely specified by the geometry and only by the latter, Reichenbach mistakenly agrees [68, p. 52; 74, p. 35] that the geom-

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eometry is sufficient to define congruence and contests only Dingler’s further claim that it is necessary.26

Carnap [8, p. 54] discusses the dependencies obtaining between (a) the metric geometry, which he symbolizes by “R” in this German publication; (b) the topology of the space and the facts concerning the coincidences of the rod in it, symbolized by “T” for “Tatbestand”; and (c) the metric M (“Mass-setzung”), which entails a congruence definition and is given by the function f (and by the choice of a unit), as will be recalled from the beginning of our Section 3.27 And he concludes that the functional relations between R, M, and T are such “that if two of them are given, the third specification is thereby uniquely given as well” [8, p. 54]. Accordingly, he writes:

\[
\begin{align*}
R &= \phi_1(M, T) \\
M &= \phi_2(R, T) \\
T &= \phi_3(M, R).
\end{align*}
\]

While the first of these dependencies does hold, our example of imparting a Euclidean geometry to a table top by each of two incompatible congruence definitions shows that not only the second but also the third of Carnap’s dependencies fails to hold. For the mere specification of M to the effect that the rod will be called congruent to itself everywhere and of R as Euclidean does not tell us whether the coincidence behavior T of the rod on the table top will be such as to coincide successively with those intervals that are equal according to the formula \(ds^2 = dx^2 + dy^2\) or with the different intervals that are equal on the basis of one of the metrizations \(ds^2 = \sec^2 \theta \, dx^2 + dy^2\) (where \(\sec^2 \theta \neq 1\)). In other words, the stated specifications of M and R do not tell us whether the rod behaves on the table top as we know it to behave in actuality or whether it squirms in any one of infinitely many different ways as compared to its actual behavior.

3. As a corollary of our proof of the nonuniqueness of the congruence definition, we can show that the following statement by Reichenbach is false: “If we say: actually a geometry G applies but we measure a geometry G’, we define at the same time a force F which causes the difference between G and G’” [72, p. 27]. Using our previous notation, we note first that instead of determining a metric tensor \(g_{ik}\) uniquely (up to an arbitrary constant), the geometry \(G\) determines an infinite class \(a\) of such tensors differing other than by being proportional to one another. But since \(F_{ik} = g_{ik} - g_{ik}'\) (where the \(g_{ik}'\) are furnished by the rod prior to its being regarded as “deformed” by any universal forces), the failure of \(G\) to determine a tensor \(g_{ik}\) uniquely (up to an arbitrary constant) issues in there being as many different universal forces \(F_{ik}\) as there are different tensors \(g_{ik}\) in the class \(a\) determined by \(G\). We see, therefore, that contrary to Reichenbach, there are infinitely many different ways in which the measuring rod can be held to be “deformed” while furnishing the same geometry \(G\).

4. Some Chronometric Ramifications of the Conventionality of Congruence

(i) Newtonian mechanics.

On the conception of time congruence as conventional, the preference for the customary definition of isochronism—a preference not felt by Einstein in the general theory of relativity, as we shall see in part (ii)—can derive only from considerations of convenience and elegance so long as the resulting form of the theory is not prescribed. Hence, the thesis that isochronism is conventional precludes a difference in factual import (content) or in explanatory power between two descriptions one of which employs the customary isochronism while the other is a “translation” (transcription) of it into a language employing a time congruence incompatible with the customary one.

As a test case for this thesis of explanatory parity, the following kind of counterargument has been suggested in outline. On the Riemannian analysis, congruence must be regarded as conventional in the time continuum of Newtonian dynamics no less than in the theory of relativity. We shall therefore wish to compare in regard to explanatory capability the two forms of Newtonian dynamics corresponding to two different time congruences as follows.

The first of these congruences is defined by the requirement that Newton’s laws hold, as modified by the addition of very small corrective terms expressing the so-called relativistic motion of the perihelia. This time
congruence will be called “Newtonian,” and the time variable whose values represent Newtonian time after a particular unit has been chosen will be denoted by “t.” The second time congruence is defined by the rotational motion of the earth. It does not matter for our purpose whether we couple the latter congruence with a unit given by the mean solar second, which is the 1/86400 part of the mean interval between two consecutive meridian passages of the fictitious mean sun, or with a different unit given by the sidereal day, which is the interval between successive meridian passages of a star. What matters is that both the mean solar second and the sidereal day are based on the periodicities of the earth’s rotational motion. Assume now that one or another of these units has been chosen, and let T be the time variable associated with that metrization, which we shall call “diurnal time.” The important point is that the time variables t and T are nonlinearly related and are associated with incompatible definitions of isochronism, because the speed of rotation of the earth varies relatively to the Newtonian time metric in several distinct ways [13, pp. 264–267]. Of these, the best known is the relative slowing down of the earth’s rotation by the tidal friction between the water in the shallow seas of the earth and the land under it. Upon calculating the positions of the moon, for example, via the usual theory of celestial mechanics, which is based on the Newtonian time metric, the observed positions of the moon in the sky would be found to be ahead of the calculated ones if we were to identify the time defined by the earth’s rotation with the Newtonian time of celestial mechanics. And the same is true of the positions of the planets of the solar system and of the moons of Jupiter in amounts all corresponding to a slowing down on the part of the earth.

Now consider the following argument for the lack of explanatory parity between the two forms of the dynamical theory respectively associated with the t and T scales of time: “Dynamical facts will discriminate in favor of the t scale as opposed to the T scale. It is granted that it is kine-

matically equivalent to say

(a) the earth’s rotational motion has slowed down relatively to
the “clocks” constituted by various revolving planets and
satellites of the solar system,
or

(b) the revolving celestial bodies speed up their periodic motions relatively to the earth’s uniform rotation.

But these two statements are not on a par explanatory in the context of the dynamical theory of the motions in the solar system. For whereas the slowing down of the earth’s rotation in formulation (a) can be understood as the dynamical effect of nearby masses (the tidal waters and their friction), no similar dynamical cause can be supplied for the accelerations in formulation (b). And the latter fact shows that a theory incorporating formulation (a) has greater explanatory power or factual import than a theory containing (b).” In precisely this vein, D’Abro, though stressing on the one hand that apart from convenience and simplicity there is nothing to choose between different metrics [16, p. 53], on the other hand addsuce the provision of causal understanding by the t-scale as an argument in its favor and thus seems to construe such differences of simplicity as involving nonequivalent descriptions:

If in mechanics and astronomy we had selected at random some arbitrary definition of time, if we had defined as congruent the intervals separating the rising and setting of the sun at all seasons of the year, say for the latitude of New York, our understanding of mechanical phenomena would have been beset with grave difficulties. As measured by these new temporal standards, free bodies would no longer move with constant speeds, but would be subjected to periodic accelerations for which it would appear impossible to ascribe any definite cause, and so on. As a result, the law of inertia would have to be abandoned, and with it the entire doctrine of classical mechanics, together with Newton’s law. Thus a change in our understanding of congruence would entail far-reaching consequences.

Again, in the case of the vibrating atom, had some arbitrary definition of time been accepted, we should have had to assume that the same atom presented the most capricious frequencies. Once more it would have been difficult to ascribe satisfactory causes to these seemingly haphazard fluctuations in frequency; and a simple understanding of the most fundamental optical phenomena would have been well-nigh impossible [16, p. 78, my italics].

To examine this argument, let us set the two formulations of dynamics corresponding to the t and T scales respectively before us mathematically in order to have a clearer statement of the issue.

The differences between the two kinds of temporal congruence with which we are concerned arise from the fact that the functional relation-

ship

T = f(t)

relating the two time scales is nonlinear, so that time intervals which are
congruent on the one scale are generally incongruent on the other. It is clear that this function is monotone increasing, and thus we know that permanently
\[ \frac{dT}{dt} \neq 0. \]

Moreover, in view of the nonlinearity of \( T = f(t) \), we know that \( dT/dt \) is not constant. Since the function \( f \) has an inverse, it will be possible to translate any set of laws formulated on the basis of either of the two time scales into the corresponding other scale. In order to see what form the customary Newtonian force law assumes in diurnal time, we must express the acceleration ingredient in that law in terms of diurnal time. But in order to derive the transformation law for the accelerations, we first treat the velocities. By the chain rule for differentiation, we have, using \( 'r' \) to denote the position vector
\[ \frac{dr}{dt} = \frac{dT}{dt} \cdot \frac{dT}{dr} \]
Suppose a body is at rest in the coordinate system in which \( r \) is measured, when Newtonian time is employed, then this body will also be held to be at rest diurnally: since we saw that the second term on the right-hand side of equation (1) cannot be zero, the left-hand side of (1) will vanish if and only if the first term on the right-hand side of (1) is zero. Though rest in a given frame in the \( T \)-scale will correspond to rest in that frame in the \( T \)-scale as well, equation (1) shows that the constancy of the non-vanishing Newtonian velocity \( dr/dt \) will not correspond to a constant diurnal velocity \( dr/dT \), since the derivative \( dT/dt \) changes with both Newtonian and diurnal time. Now, differentiation of equation (1) with respect to the Newtonian time \( t \) yields
\[ \frac{d^2r}{dt^2} = \frac{dr}{dt} \frac{d^2T}{dt^2} + \frac{dT}{dt} \frac{d^2T}{dr} \left( \frac{dr}{dT} \right). \]
But, applying the chain-rule to the second factor in the second term on the right-hand side of (2), we obtain
\[ \frac{d}{dt} \left( \frac{dr}{dT} \right) = \frac{d^2r}{dT^2} \frac{dT}{dt}. \]
Hence (2) becomes
\[ \frac{d^2r}{dt^2} = \frac{dr}{dT} \frac{d^2T}{dt^2} + \frac{d^2r}{dT^2} \left( \frac{dT}{dt} \right)^2. \]

Solving for the diurnal acceleration, and using equation (1) as well as the abbreviations
\[ f'(t) = \frac{dT}{dt} \quad \text{and} \quad f''(t) = \frac{d^2T}{dt^2}, \]
we find
\[ \frac{d^2r}{dt^2} = \frac{1}{[f'(t)]^2} \frac{d^2r}{dT^2} \left( \frac{f''(t)}{[f'(t)]^3} \frac{dT}{dt} \right), \]

Several ancillary points should be noted briefly in regard to equation (4) before seeing what light it throws on the form assumed by causal explanation within the framework of a diurnal description. When the Newtonian force on a body is not zero because the body is accelerating under the influence of masses, the diurnal acceleration will generally also not be zero, save in the unusual case when
\[ \frac{d^2r}{dt^2} = \frac{f''(t)}{f'(t)} \frac{dr}{dt}. \]

Thus the causal influence of masses, which gives rise to the Newtonian accelerations in the usual description, is seen in (4) to make a definite contribution to the diurnal acceleration as well. But the new feature of the diurnal description of the facts lies in the possession of a secular acceleration by all bodies not at rest, even when no masses are inducing Newtonian accelerations, so that the first term on the right-hand side of (4) vanishes. And this secular acceleration is numerically not the same for all bodies but depends on their velocities \( dr/dt \) in the given reference frame and thus also on the reference frame.

The character and existence of this secular acceleration calls for several kinds of comment.

Its dependence on the velocity and on the reference frame should neither occasion surprise nor be regarded as a difficulty of any sort. As to the velocity dependence of the secular acceleration, consider a simple numerical example which removes any surprise: if instead of calling two successive hours on Big Ben equal, we remetrized time so as to assign the measure \( \frac{1}{2} \) hour to the second of these intervals, then all bodies having uniform
speeds in the usual time metric will double their speeds on the new scale after the first interval, and the numerical increase or acceleration in the speeds of initially faster bodies will be greater than that in the speeds of the initially slower bodies. Now as for the dependence of the secular acceleration on the reference frame, in the context of the physical facts asserted by the Newtonian theory apart from its metrical philosophy it is a mere prejudice to require that, to be admissible, a formulation of that theory must agree with the customary one in making the acceleration of a body at any given time be the same in all Galilean reference frames ("Galilean relativity"). For not a single bona fide physical fact of the Newtonian world is overlooked or contradicted by a kinematics not featuring this Galilean relativity. It is instructive to be aware in this connection that even in the customary rendition of the kinematics of special relativity, a constant acceleration in a frame S' would not generally correspond to a constant acceleration in a frame S, because the component accelerations in S depend not only on the accelerations in S' but also on the component velocities in that system which would be changing with the time.

But what are we to say, apart from the dependence on the velocity and reference system, about the very presence of this "dynamically unexplained" or causally baffling secular acceleration? To deal with this question, we first observe merely for comparison that in the customary formulation of Newtonian mechanics, constant speeds (as distinct from constant velocities) fall into two classes with respect to being attributable to the dynamical action of perturbing masses: constant rectilinear speeds are affirmed to prevail in the absence of any mass influences, while constant curvilinear (e.g., circular) speeds are related to the (centripetally) accelerating actions of masses. Now in regard to the presence of a secular acceleration in the diurnal description, it is fundamental to see the following: Whereas on the version of Newtonian mechanics employing the customary metrizations (of time and space) all accelerations whatsoever in Galilean frames are of dynamical origin by being attributable to the action of specific masses, this feature of Newton's theory is made possible not only by the facts but also by the particular time metrization chosen to codify them. As equation (4) shows upon equating \( \frac{d^2r}{dt^2} \) to zero, the dynamical character of all accelerations is not vouchsafed by any causal facts of the world with which every theory would have to come to terms. For the diurnal description encompasses the objective behavior of bodies (point events and coincidences) as a function of the presence or absence

of other bodies no less than does the Newtonian one, thereby achieving full explanatory parity with the latter in all logical (as distinct from pragmatic!) respects.

Hence the provision of a dynamical basis for all accelerations should not be regarded as an inflexible epistemological requirement in the elaboration of a theory explaining mechanical phenomena. Disregarding the pragmatically decisive consideration of convenience, there can therefore be no valid explanatory objection to the diurnal description, in which accelerations fall into two classes by being the superpositions, in the sense of equation (4), of a dynamically grounded and a kinematically grounded term. And, most important, since there is no slowing down of the earth's rotation on the diurnal metric, there can be no question in that description of specifying a cause for such a nonexistent deceleration; instead, a frictional cause is now specified for the earth's diurnally uniform rotation and for the liberation of heat accompanying this kind of uniform motion. For in the T-scale description it is uniform rotation which requires a dynamical cause constituted by masses interacting (frictionally) with the uniformly rotating body, and it is now a law of nature or entailed by such a law that all diurnally uniform rotations issue in the dissipation of heat.

Of course, the mathematical representation of the frictional interaction will not have the customary Newtonian form: to obtain the diurnal account of the frictional dynamics of the tides, one would need to apply transformations of the kind given in our equation (4) to the quantities appearing in the relevant Newtonian equations for this case, which can be found in [43, Ch. 8] and in [88].

But, it will be asked, what of the Newtonian conservation principles, if the T scale of time is adopted? It is readily demonstrable by reference to the simple case of the motion of a free particle that while the Newtonian kinetic energy will be constant in this case, its formal diurnal homologue (as opposed to its diurnal equivalent!) will not be constant. Let us denote the constant Newtonian velocity of the free particle by \( v_b \), the subscript "t" serving to represent the use of the t scale, and let \( v_T \) denote the diurnal velocity corresponding to \( v_b \). Since we know from equation (1) above that \( v_t = v_T dT/dt \), where \( v_t \) is constant but \( dT/dt \) is not, we see that the diurnal homologue \( \frac{1}{2}mv_T^2 \) of the Newtonian kinetic energy cannot be constant in this case, although the diurnal equivalent \( \frac{1}{2}mv_T^2 \) is necessarily constant. Just as in the case of the Newtonian equations of
motion themselves, so also in the case of the Newtonian conservation principle of mechanical energy, the diurnal equivalent or transcription explains all the facts explained by the Newtonian original. Hence our critic can derive no support at all from the fact that the formal diurnal homologues of Newtonian conservation principles generally do not hold. And we see, incidentally, that the time invariance of a physical quantity and hence the appropriateness of singling it out from among others as a form of "energy," etc., will depend not only on the facts but also on the time metrization used to render them. It obviously will not do, therefore, to charge the diurnal description with inconsistency via the petitio of grafting onto it the requirement that it incorporate the homologues of Newtonian conservation principles which are incompatible with it: a case in point is the charge that the diurnal description violates the conservation of energy because in its metric the frictional generation of heat in the tidal case is not compensated by any reduction in the speed of the earth’s rotation! Whether the diurnal time metrization permits the deduction of conservation principles of a relatively simple type involving diurnally based quantities is a rather involved mathematical question whose solution is not required to establish our thesis that, apart from pragmatic considerations, the diurnal description enjoys explanatory parity with the Newtonian one.

We have been disregarding pragmatic considerations in assessing the explanatory capabilities of two descriptions associated with different time metrizations as to parity. But it would be an error to infer that in pointing to the equivalence of such descriptions in regard to factual content, we are committed to the view that there is no criterion for choosing between them and hence no reason for preferring any one of them to the others. Factual adequacy (truth) is, of course, the cardinal necessary condition for the acceptance of a scientific theory, but it is hardly a sufficient condition for accepting any one particular formulation of it which satisfies this necessary condition. As well say that a person pointing out that equivalent descriptions can be given in the decimal (metric) and English system of units cannot give telling reasons for preferring the former! Indeed, after first commenting on the factual basis of the existence of the Newtonian time congruence, we shall see that there are weighty pragmatic reasons for preferring that metrization of the time continuum. And these reasons will turn out to be entirely consonant with our twin contention that alternative metrizability allows linguistically different, equivalent descriptions and that geochronometric conventionalism is not a subthesis of trivial semantical conventionalism.

The factual basis of the Newtonian time metrization will be appreciated by reference to the following two considerations: (i) As we shall prove presently, it is a highly fortunate empirical fact, and not an a priori truth, that there exists a time metrization at all in which all accelerations with respect to inertial systems are of dynamic origin, as claimed by the Newtonian theory, and (ii) it is a further empirical fact that the time metrization having this remarkable property (i.e., "ephemeris time") is furnished physically by the earth’s annual revolution around the sun (not by its diurnal rotation) albeit not in any observationally simple way, since due account must be taken computationally of the irregularities produced by the gravitational influences of the other planets. ²⁸ That the existence of a time metrization in which all accelerations with respect to inertial systems are of dynamical origin cannot be guaranteed a priori is demonstrable as follows.

Suppose that, contrary to actual fact, it were the case that a free body did accelerate when its motion is described in the metric of ephemeris time t, it thus being assumed that there are accelerations in the customary time metric which are not dynamical in origin. More particularly, let us now posit that, contrary to actual fact, a free particle were to execute one-dimensional simple harmonic motion of the form

\[ r = \cos at, \]

where r is the distance from the origin. In that hypothetical eventuality, the acceleration of a free particle in the t scale would have the time-dependent value

\[ \frac{d^2r}{dt^2} = -a^2 \cos at. \]

And our problem is to determine whether there would then exist some other time metrization \( T = f(t) \) possessing the Newtonian property that our free particle has a zero acceleration. We shall now find that the answer is definitely negative: under the hypothetical empirical conditions which we have posited, there would indeed be no admissible, single-valued

²⁸ For details on the so-called ephemeris time given by this metric, see [10, 11, 12, 13].
time metrization T at all in which all accelerations with respect to inertial systems would be of dynamical origin.

For let us now regard "T" in equation (4) of this subsection as the time variable associated with the sought-after metrization T = f(t) in which the acceleration d²r/dT² of our free particle would be zero. We recall that equation (5) of this subsection was obtained from equation (4) by equating the T-scale acceleration d²r/dT² to zero. Hence if our sought-after metrization exists at all, it would have to be the solution T = f(t) of the scalar form of equation (5) as applied to our one-dimensional motion. That equation is

\[
\frac{d^2r}{dt^2} = \frac{f''(t)}{f(t)} \frac{dr}{dt}.
\]

Putting v = dr/dt and noting that

\[
\frac{d}{dt} \log f(t) = \frac{f''(t)}{f(t)} \text{, and } \frac{d}{dt} \log v = \frac{1}{v} \frac{dv}{dt} \text{,}
\]

equation (6) becomes

\[
\frac{d}{dt} \log v = \frac{d}{dt} \log f(t).
\]

Integrating, and using log c as the constant of integration, we obtain

\[
\log v = \log cf(t),
\]

or

\[
v = cf(t),
\]

which is

\[
\frac{dr}{dt} = \frac{dT}{dt}.
\]

Integration yields

\[
r = cT + d,
\]

where d is a constant of integration. But, by our earlier hypothesis, r = \cos\omega t. Hence (7) becomes

\[
T = \frac{1}{c} \cos\omega t - \frac{d}{c}.
\]

It is evident that the solution T = f(t) given by equation (8) is not a one-to-one function: the same time T in the sought-after metrization would correspond to all those different times on the t scale at which the oscillating particle would return to the same place r = \cos\omega t in the course of its periodic motion. And by thus violating the basic topological requirement that the function T = f(t) be one-to-one, the T scale which does have the sought-after Newtonian property under our hypothetical empirical conditions is physically quite inadmissible and hence unavailable metrization.

It follows that there is no a priori assurance of the existence of at least one time metrization possessing the Newtonian property that the acceleration of a free particle in inertial systems is zero. So much for the factual basis of the Newtonian time metrization.

Now, inasmuch as the employment of the time metrization based on the earth's annual revolution issues in Newton's relatively simple laws, there are powerful reasons of mathematical tractability and convenience for greatly preferring the time metrization in which all accelerations are of dynamical origin. In fact, the various refinements which astronomers have introduced in their physical standards for temporal congruence have been dictated by the demand for a definition of temporal congruence (or of a so-called invariable time standard) for which Newton's laws will hold in the solar system, including the relatively simple conservation laws interconnecting diverse kinds of phenomena (mechanical, thermal, etc.). And thus, as Feigl and Maxwell have aptly put it, one of the important criteria of descriptive simplicity which greatly restrict the range of "reasonable" conventions is seen to be the scope which a convention will allow for mathematically tractable laws.


In the special theory of relativity, only the customary time metrization is employed in the following sense: At any given point A in a Galilean frame, the length of a time interval between two events at the point A is given by the difference between the time coordinates of the two events as furnished by the readings of a standard clock at A whose periods are defined to be congruent. This is, of course, the precise analogue of the customary definition of spatial congruence which calls the rod congruent to itself everywhere, when at relative rest, after allowance for substance-specific perturbations. On the other hand, as we shall now see, there are contexts in which the general theory of relativity (GTR) utilizes a criterion of temporal congruence which is an analogue of a noncustomary kind of spatial congruence in the following sense: the length of a time
interval separating two events at a clock depends not only on the difference between the time coordinates which the clock assigns to these events but also on the spatial location of the clock (though not on the time itself at which the interval begins or ends).

A case in point from the GTR involves a rotating disk to which we apply those principles which GTR takes over from the special theory of relativity. Let a set of standard material clocks be distributed at various points on such a disk. The infinitesimal application of the special relativity clock retardation then tells us the following: a clock at the center O of the disk will maintain the rate of a contiguous clock located in an inertial system I with respect to which the disk has angular velocity \( \omega \), but the same does not hold for clocks located at other points A of the disk which are at positive distances \( r \) from O. Such A clocks have various linear velocities \( \omega r \) relatively to I in virtue of their common angular velocity \( \omega \). Accordingly, all A clocks (whatever their chemical constitution) will have readings lagging behind the corresponding readings of the respective I-system clocks adjacent to them by a factor of

\[
\sqrt{1 - \frac{r^2 \omega^2}{c^2}},
\]

where \( c \) is the velocity of light. What would be the consequences of using the customary time metrization everywhere on the rotating disk and letting the duration (length) of a time interval elapsing at a given point A be given by the difference between the time coordinates of the termini of that interval as furnished by the readings of the standard clock at A? The adoption of the customary time metric would saddle us with a most complicated description of the propagation of light in the rotating system having the following undesirable features: (i) time would enter the description of nature explicitly in the sense that the one-way velocity of light would depend on the time, since the lagging rate of the clock at A issues in a temporal change in the magnitude of the one-way transit time of a light ray for journeys between O and A, and (ii) the number of light waves emitted at A during a unit of time on the A clock is greater than the number of waves arriving at the center O in one unit of time on the O clock [53, pp. 225–226]. To avoid the undesirably complicated laws entailed by the use of the simple customary definition of time congruence, the GTR jettisoned the latter. In its stead, it adopted the following more complicated, noncustomary congruence definition for the sake of the simplicity of the resulting laws: at any point A on the disk the length (duration) of a time interval is given not by the difference between the A clock coordinates of its termini but by the product of this increment and the rate factor

\[
\frac{1}{\sqrt{1 - \frac{r^2 \omega^2}{c^2}}},
\]

which depends on the spatial coordinate \( r \) of the point A. This rate factor serves to assign a greater duration to time intervals than would be obtained from the customary procedure of letting the length of time be given by the increment in the clock readings. In view of the dependence of the metric on the spatial position \( r \), via the rate factor entering into it, we are confronted here with a noncustomary time metrization fully as consonant with the temporal order of the events at A as is the customary metric.

A similarly nonstandard time metric is used by Einstein in his GTR paper of 1911 [24, Section 3] in treating the effect of gravitation on the propagation of light. Analysis shows that the very same complexities in the description of light propagation which are encountered on the rotating disk arise here as well, if the standard time metric is used. These complexities are eliminated here in quite analogous fashion by the use of a noncustomary time metric. Thus, if we are concerned with light emitted on the sun and reaching the earth and if \( "-\Phi" \) represents the negative difference in gravitational potential between the sun and the earth, then we proceed as follows: prior to being brought from the earth to the sun, a clock is set to have a rate faster than that of an adjoining terrestrial clock by a factor of

\[
\frac{1}{1 - \frac{\Phi}{c^2}}
\]

(to a first approximation), where

\[
\frac{\Phi}{c^2} < 1.
\]

(iii) The Cosmology of E. A. Milne.

E. A. Milne, whose two logarithmically related \( t \) and \( \tau \) scales of time were mentioned in Section 2, part (i), has attempted to erect the usual
space-time structure of special relativity on the basis of a light signal
kinematics of particle observers purportedly dispensing with the use of
rigid solids and isochronous material clocks [51, 52]. In his Modern Cosmo-
logy and the Christian Idea of God [52, Ch. III], Milne begins his
discussion of time and space by incorrectly charging Einstein with failure
to realize that the concept of a rigid body as a body whose rest length is
invariant under transport contains a conventional ingredient just as much
as does the concept of metrical simultaneity at a distance.9 Milne then
proposes to improve upon a rigid body criterion of spatial congruence by
proceeding in the manner of radar ranging and using instead the round-
trip times required by light to traverse the corresponding closed paths,
these times not being measured by material clocks but, in outline, as fol-
lows.80 Each particle is equipped with a device for ordering the genidienti-
cal events belonging to it temporally in a linear Cantorian continuum.
Such a device is called a "clock," and the single observer at the particle
using such a local clock is called a "particle observer." If now A and B are
two particle observers and light signals are sent from one to the other, then
the time of arrival \( t' \) at B can be expressed as a function \( t' = f(t) \) of the
time \( t \) of emission at A, and likewise the time of arrival \( t' \) at A is a func-
tion \( t = F(t') \) of the time \( t' \) of emission at B. Particle observers equipped
with clocks as defined are said to be "equivalent," if the so-called signal
functions \( f \) and \( F \) are the same, and the clocks of equivalent particle
observers are said to be congruent. It can be shown that if A and B are not
equivalent, then B's clock can be regraduated by a transformation of the form
\( t' = \psi(t) \) so as to render them equivalent [52, pp. 39-41]. The con-
gruence of the clocks at A and B does not, of course, assure their synchro-
nism. Milne now uses Einstein's definition of simultaneity [52, p. 42]:
the time \( t_2 \) assigned by A to the arrival of a light ray at B which is emitted
at time \( t_1 \) at A and returns to A at time \( t_3 \) after instantaneous reflection
at B is defined to be

\[
t_2 = \frac{1}{2} (t_1 + t_3).
\]

And he defines the distance \( r_2 \) of B, by A's clock, upon the arrival of the
light from A at B to be given by the relation

\[
r_2 = \frac{1}{2} c(t_3 - t_1),
\]

where \( c \) is an arbitrarily chosen constant [52, p. 42]. Since

\[
\frac{r_2}{t_2 - t_1} = \frac{r_2}{t_3 - t_2} = c,
\]

the constant \( c \) represents the velocity of the light signal in terms of the
conventions adopted by A for measuring distance and time at a remote
point B. Milne gives the following statement of his epistemological ob-
jections to Einstein's use of rigid rods and of his claim that his light-signal
kinematics provides a philosophically satisfactory alternative to it:

... the concept of the transport of a rigid body or rigid length measure
is itself an indefinable concept. In terms of one given standard metre, we
cannot say what we mean by asking that a given 'rigid' length measure
shall remain 'unaltered in length' when we move it from one place to anoth-
er; for we have no standard of length at the new place. Again, we
should have to specify standards of 'rest' everywhere, for it is not clear
without consideration that the 'length' will be the same, even at the same
place, for different velocities. The fact is that to say of a body or measur-
ring-rod that it is 'rigid' is no definition whatever; it specifies no 'opera-
tional' procedure for testing whether a given length-measure after trans-
port or after change of velocity is the same as it was before [52, p. 35].

It is part of the debt we owe to Einstein to recognize that only 'opera-
tional' definitions are of any significance in science. Einstein carried
out his own procedure completely when he analysed the previously un-
defined concept of simultaneity, replacing it by tests using the measure-
ments which have actually to be employed to recognize whether two dis-
tant events are or are not simultaneous. But he abandoned his own pro-
cedure when he retained the indefinable concept of the length of a 'rigid'
body, i.e. a length unaltered under transport. The two indefinable con-
cepts of the transportable rigid body and of simultaneity are on exactly
the same footing; they are fog-centres, inhibiting further vision, until
analysed and shown to be equivalent to conventions [52, p. 35].

It will be one of our major tasks to elucidate the type of graduation
employed for graduating our ordinary clocks; that is to say, to inquire
what is meant by, and if possible to isolate what is usually understood by,
'uniform time'. In other words, we wish to inquire which of the arbitrar-
ily many ways in which the markings of our abstract clock may be gradu-
can be identified with the 'uniform time' of physics [52, p. 37].

The question now arises: is it possible to arrange that the mode of
graduation of observer B's clock corresponds to the mode of graduation
of A's clock in such a way that a meaning can be attached to saying that
B's clock is a copy of A's clock? If so, we shall say that B's clock has been
made congruent with A's [52, p. 39].

It will have been noticed that we have succeeded in making B's clock
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a copy of A’s without bringing B into permanent coincidence with A. We have made a copy of an arbitrary clock at a distance. This is something we cannot do with metre-scales or other length-measures. The problem of copying a clock is in principle simpler than the problem of copying a unit of length. We shall see in due course that with the construction of a copy of a clock at a distance we have solved the problem of comparing lengths [52, p. 41].

The important point is that epoch and distance (which we shall call coordinates) are purely conventional constructs, and have meaning only in relation to a particular form of clock graduation . . . But it is to be pointed out that when the mode of clock graduation reduces to that of ordinary clocks in physical laboratories, our coordinate conventions provide measures of epoch and distance which coincide with those based on the standard metre [52, pp. 42–43].

The reason why it is more fundamental to use clocks alone rather than both clocks and scales or than scales alone is that the concept of the clock is more elementary than the concept of the scale. The concept of the clock is connected with the concept of ‘two times at the same place’, whilst the concept of the scale is connected with the concept of ‘two places at the same time’. But the concept of ‘two places at the same time’ involves a convention of simultaneity, namely, simultaneous events at the two places, but the concept of ‘two times at the same place’ involves no convention; it involves only the existence of an ego [52, p. 46].

Length is just as much a conventional matter as an epoch at a distance. Thus the metre-scale is not such a fundamental instrument as the clock. In the first place its length for any observer, as measured by the radar method, depends on the clock used by the observer; in the second place, different observers assign different lengths to it even if their clocks are congruent, owing to the fact that the test of simultaneity is a conventional one. The clock, on the other hand, once graduated, gives epochs at itself which are independent of convention.

Once we have set up a clock, arbitrarily graduated, distances for the observer using this clock become definite. If a rod, moved from one position of rest relative to this observer to another position of rest relative to the same observer, possesses in the two positions the same length, as measured by this observer using his own clock, as graduated, then the rod is said to have undergone a rigid-body-displacement by this clock. In this way we see that once we have fixed on a clock, a rigid-body-displacement becomes definable. But until we have provided a clock, there is no way of saying what we mean by a rigid body under displacement [52, pp. 47–48].

Now, if Milne is to make good his criticism of Einstein by erecting the space-time structure of special relativity on alternative epistemological foundations, he must provide us with inertial systems by means of the re-

sources of his light-signal kinematics as well as with the measures of length and time on which the kinematics of special relativity is predicated. This means that he must be able to characterize inertial systems within the confines of his epistemological program as some kind of dense assemblage of equivalent particle observers filling space such that each particle observer is at rest relative to and synchronous with every other. We have already seen that he was wholly in error in charging Einstein with lack of awareness of the conventionality of spatial congruence as defined by the rigid rod. But that, much more fundamentally, he is mistaken in believing to have erected the kinematics of special relativity on an epistemologically more satisfactory base than Einstein did will now be made clear by reference to the following result pointed out by L. L. Whyte [102]: Using only light signals and temporal succession without either a solid rigid rod or an isochronous material clock, it is not possible to construct ordinary measures of length and time. For “a physicist using only light signals cannot discriminate inertial systems from these subjected to arbitrary 4-D similarity transformations.” The system of ‘resting’ mass points which can be so identified may be arbitrarily expanding and/or contracting relatively to a rod, and these superfluous transformations can only be eliminated by using a rod or a clock” [102, p. 161].

The significance of the result stated by Whyte is twofold: (i) If Milne dispenses with material clocks and bases his chronometry only on the congruences yielded by his light-signal clocks, then he cannot obtain inertial systems without a rigid rod in the following sense. The rigid rod is not needed for the definition of spatial congruence within the system but is required to assure that the distance between one particular pair of points connected by it at one time $t_0$ is the same as at some later time $t_1$. In other words, the rod is rigid at a given place by remaining congruent to itself (by convention) as time goes on. And in this way the rod assures the time constancy of the distance between the two given points connected by it. This reliance on the rigid rod thus involves the use of the definition of simultaneity. Hence, if Milne were right in charging that the use of a rigid rod is beset by philosophic difficulties, then he indeed would be incurring these liabilities no less than Einstein does. On the other hand, suppose that (ii) Milne does use a material clock to define the time metric at a space point and thereby to particularize his clock graduations

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80 For a brief account of similarity transformations, and a further articulation of Whyte’s point here, cf. [72, pp. 172–173].

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to the kind required for the elimination of the unwanted reference systems described by L. L. Whyte. This procedure is a far cry from his purely topological clock which "involves only the existence of an ego" [52, p. 46] in contradistinction to the rigid scale's involvement of a definition of simultaneity. And, in that case, his measurement of the equality of space intervals by means of the equality of the corresponding round-trip time intervals involves the following conventions: (a) the tacit use of a definition of simultaneity of noncoinciding events. For although a round-trip time on a given clock does not, of course, itself require such a simultaneity criterion, the measurement of a spatial distance in an inertial system by means of this time does: the distance yielded by the round-trip time on a clock at A is the distance \( r_2 \) between A and B at the time \( t_2 \) on the A clock when the light pulse from A arrives at B on its round-trip ABA, (b) successive equal differences in the readings of a given local clock are stipulated to be measures of equal time intervals and thereby of equal space intervals, and (c) equal differences on separated clocks of identical constitution are decreed to be measures of equal time intervals and thereby of equal space intervals.

To what extent then, if any, does Milne have a case against Einstein? It would appear from our analysis that the only justifiable criticism is not at all epistemological but concerns an innocuous point of axiomatic economy: once you grant Milne a material clock, he does not require the rigid rod at all, whereas Einstein utilizes the spatial congruence definition based on the rigid rod in addition to all of the conventions needed by Milne. Thus, Milne's kinematics, as supplemented by the use of a material clock, is constructed on a slightly narrower base of conventions than is Einstein's.82

It will be recalled that if measurements of spatial and temporal extension are to be made by means of solid rods and material clocks, allowance must be made computationally for thermal and other perturbations of these bodies so that they can define rigidity and isochronism. Calling attention to this fact and believing Milne's light-signal kinematics to be essentially successful, L. Page [59, pp. 78–79] deemed Milne's construction more adequate than Einstein's, writing: "the original formulation of the relativity theory was based on undefined concepts of space and time intervals which could not be identified unambiguously with actual observations. Recently Milne has shown how to supply the desired criterion [of rigidity and isochronism] by erecting the space-time structure on the foundations of a constant light-signal velocity." It is apparent in the light of our appraisal of Milne's kinematics that Page's claim is vitiated by Milne's need for a rigid rod or material clock as specified.

It should be noted, however, as Professor A. G. Walker has pointed out to me, that if Milne's construction is interpreted as applying not to special relativity kinematics but to his cosmological world model, then our criticisms are no longer pertinent. In terms of his logarithmically related \( \tau \) and t scales of time, it turns out that upon measuring distances by the specified chronometric convention, the galaxies are at relative rest in \( \tau \)-scale kinematics and in uniform relative motion in the t scale. Each of these time scales is unique up to a trivial change of units, and their associated descriptions of the cosmological world are equivalent in Reichenbach's sense. In this cosmological context, the problem of eliminating the superfluous transformations mentioned by Whyte therefore does not arise.

5. Critique of Some Major Objections to the Conventionality of Spatio-Temporal Congruence

(i) The Russell-Poincaré Controversy.

During the years 1897–1900, B. Russell and H. Poincaré had a controversy which was initiated by Poincaré's review [62] of Russell's Foundations of Geometry of 1897, and pursued in Russell's reply [81] and Poincaré's rejoinder [65]. Russell criticized Poincaré's conventionalist conception of congruence and invoked the existence of an intrinsic metric as follows:83

It seems to be believed that since measurement [i.e., comparison by means of the congruence standard] is necessary to discover equality or inequality, these cannot exist without measurement. Now the proper conclusion is exactly the opposite. Whatever one can discover by means of an operation must exist independently of that operation: America existed before Christopher Columbus, and two quantities of the same kind must be equal or unequal before being measured. Any method of measurement [i.e., any congruence definition] is good or bad according as it yields a result which is true or false. Mr. Poincaré, on the other hand, holds that

82 The preceding critique of Milne supplants my earlier brief critique [36, pp. 531–533] in which Milne's arguments were misinterpreted as indicative of lack of appreciation on his part of the conventionality of temporal congruence.

83 This argument is implicitly endorsed by Helmholtz [91, p. 15].
measurement creates equality and inequality [i.e., that there is no intrinsic metric]. It follows [then] . . . that there is nothing left to measure and that equality and inequality are terms devoid of meaning [81, pp. 687–688].

We have argued that the Newtonian position espoused by Russell is untenable. But our critique of the model-theoretic trivialization of the conventionality of congruence shows that we must reject as inadequate the following kind of criticism of Russell’s position, which he would have regarded as a petitio principii: “Russell’s claim is an absurdity, because it is the denial of the truism that we are at liberty to give whatever physical interpretations we like to such abstract signs as ‘congruent line segments’ and ‘straight line’ and then to inquire whether the system of objects and relations thus arbitrarily named is a model of one or another abstract geometric axiom system. Hence, these linguistic considerations suffice to show that there can be no question, as Russell would have it, whether two non-coinciding segments are truly equal or not and whether measurement is being carried out with a standard yielding results that are true in that sense. Accordingly, awareness of the model-theoretic conception of geometry would have shown Russell that alternative metrizability of spatial and temporal continua should never have been either startling or a matter for dispute. And, by the same token, Poincaré could have spared himself a polemic against Russell in which he spoke misleadingly of the conventionality of congruence as a philosophical doctrine pertaining to the structure of space.”

Since this model-theoretic argument fails to come to grips with Russell’s root assumption of an intrinsic metric, he would have dismissed it as a petitio by raising exactly the same objections that the Newtonian would adduce (cf. Section 2, part (i)) against the alternative metrizability of space and time. And Russell might have gone on to point out that the model theoretician cannot evade the spatial equality issue by (i) noting that there are axiomatizations of each of the various geometries dispensing with the abstract relation term “congruent” (for line segments), and (ii) claiming that there can then be no problem as to what physical interpretations of that relation term are permissible. For a metric geometry makes metrical comparisons of equality and inequality, however covertly or circuitously these may be rendered by its language. It is quite material, therefore, whether the relation of spatial equality between line segments is designated by the term “congruent” or by some other term or

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terms. Thus, for example, Tarski’s axioms for elementary Euclidean geometry [87] do not employ the term “congruent” for this purpose, using instead a quaternary predicate denoting the equidistance relation between four points. Also, in Sophus Lie’s group-theoretical treatment of metric geometries, the congruences are specified by groups of point transformations [3, pp. 153–154]. But just as Russell invoked his conception of an intrinsic metric to restrict the permissible spatial interpretations of “congruent line segments,” so also he would have maintained that it is never arbitrary what quartets of physical points may be regarded as the denotata of Tarski’s quaternary equidistance predicate. And he would have imposed corresponding restrictions on Lie’s transformations, since the displacements defined by these groups of transformations have the logical character of spatial congruences. These considerations show that it will not suffice in this context simply to take the model-theoretic conception of geometry for granted and thereby to dismiss the Russell-Helmholtz claim peremptorily in favor of alternative metrizability. Rather what is needed is a refutation of the Russell-Helmholtz root assumption of an intrinsic metric: to exhibit the untenability of that assumption as we have endeavored to do in Section 2 is to provide the justification of the model-theoretic affirmation that a given set of physico-spatial facts may be held to be as much a realization of a Euclidean calculus as of a non-Euclidean one yielding the same topology.

The refutation presented in Section 2 requires supplementation, however, to invalidate A. N. Whitehead’s perceptualistic version of Russell’s argument. We therefore now turn to an examination of Whitehead’s philosophy of congruence.

(ii) A. N. Whitehead’s Unsuccessful Attempt to Ground an Intrinsic Metric of Physical Space and Time on the Deliverances of Sense.

Commenting on the Russell-Poincaré controversy [97, pp. 121–124], Whitehead maintains the following: (i) Poincaré’s argument on behalf of alternative metrizability is unanswerable only if the philosophy of physical geometry and chronometry is part of an epistemological framework resting on the more ultimate metric deliverances of sense perception, and (iii) perceptual time and space exhibit an
intrinsic metric. Specifically, Whitehead proposes to point out “the factor in nature which issues in the preeminence of one congruence relation over the indefinite herd of other such relations” [97, p. 124] and writes: The reason for this result is that nature is no longer confined within space at an instant. Space and time are now interconnected; and this peculiar factor of time which is so immediately distinguished among the deliverances of our sense-awareness, relates itself to one particular congruence relation in space [97, p. 124]. . . . Congruence depends on motion, and thereby is generated the connexion between spatial congruence and temporal congruence [97, p. 126].

Whitehead’s argument is thus seen to turn on his ability to show that temporal congruence cannot be regarded as conventional in physics. He believes to have justified this crucial claim by the following reasoning in which he refers to the conventionalist conception as “the prevalent view” and to his opposing thesis as “the new theory”:

The new theory provides a definition of the congruence of periods of time. The prevalent view provides no such definition. Its position is that if we take such time-measurements so that certain familiar velocities which seem to us to be uniform are uniform, then the laws of motion are true. Now in the first place no change could appear either as uniform or non-uniform without involving a definite determination of the congruence for time-periods. So in appealing to familiar phenomena it allows that there is some factor in nature which we can intellectually construct as a congruence theory. It does not however say anything about it except that the laws of motion are then true. Suppose that with some expositors we cut out the reference to familiar velocities such as the rate of rotation of the earth. We are then driven to admit that there is no meaning in temporal congruence except that certain assumptions make the laws of motion true. Such a statement is historically false. King Alfred the Great was ignorant of the laws of motion, but knew very well what he meant by the measurement of time, and achieved his purpose by means of burning candles. Also no one in past ages justified the use of sand in hour glasses by saying that some centuries later interesting laws of motion would be discovered which would give meaning to the statement that the sand was emptied from the bulbs in equal times. Uniformity in change is directly perceived, and it follows that mankind perceives in nature factors from which a theory of temporal congruence can be formed. The prevalent theory entirely fails to produce such factors [97, p. 137]. . . .

On the orthodox theory the position of the equations of motion is most ambiguous. The space to which they refer is completely undetermined and so is the measurement of the lapse of time. Science is simply setting out on a fishing expedition to see whether it cannot find some procedure which it can call the measurement of space and some procedure which it can call the measurement of time, and something which it can call a system of forces, and something which it can call masses, so that these formulæ may be satisfied. The only reason—on this theory—why anyone should want to satisfy these formulæ is a sentimental regard for Galileo, Newton, Euler and Lagrange. The theory, so far from founding science on a sound observational basis, forces everything to conform to a mere mathematical preference for certain simple formulæ.

I do not for a moment believe that this is a true account of the real status of the Laws of Motion. These equations want some slight adjustment for the new formulæ of relativity. But with these adjustments, imperceptible in ordinary use, the laws deal with fundamental physical quantities which we know very well and wish to correlate.

The measurement of time was known to all civilised nations long before the laws were thought of. It is this time as thus measured that the laws are concerned with. Also they deal with the space of our daily life. When we approach to an accuracy of measurement beyond that of observation, adjustment is allowable. But within the limits of observation we know what we mean when we speak of measurements of space and measurements of time and uniformity of change. It is for science to give an intellectual account of what is so evident in sense-awareness. It is to me thoroughly incredible that the ultimate fact beyond which there is no deeper explanation is that mankind has really been swayed by an unconscious desire to satisfy the mathematical formulæ which we call the Laws of Motion, formulæ completely unknown till the seventeenth century of our epoch [97, pp. 139–140].

After commenting that purely mathematically, an infinitude of incompatible spatial congruence classes of intervals satisfy the congruence axioms, Whitehead says:

This breakdown of the uniqueness of congruence for space . . . is to be contrasted with the fact that mankind does in truth agree on a congruence system for space and a congruence system for time which are founded on the direct evidence of its senses. We ask, why this pathetic trust in the yard measure and the clock? The truth is that we have observed something which the classical theory does not explain.

It is important to understand exactly where the difficulty lies. It is often wrongly conceived as depending on the inexactness of all measurements in regard to very small quantities. According to our methods of observation we may be correct to a hundredth, or a thousandth, or a millionth of an inch. But there is always a margin left over within which we cannot measure. However, this character of inexactness is not the difficulty in question.
Let us suppose that our measurements can be ideally exact; it will be still the case that if one man uses one qualifying [i.e., congruence] class γ and the other man uses another qualifying [i.e., congruence] class δ, and if they both admit the standard yard kept in the exchequer chambers to be their unit of measurement, they will disagree as to what other distances [at other] places should be judged to be equal to that standard distance in the exchequer chambers. Nor need their disagreement be of a negligible character [99, pp. 49-50].

When we say that two stretches match in respect to length, what do we mean? Furthermore we have got to include time. When two lapses of time match in respect to duration, what do we mean? We have seen that measurement presupposes matching, so it is of no use to hope to explain matching by measurement [99, pp. 50-51].

Our physical space therefore must already have a structure and the matching must refer to some qualifying class of quantities inherent in this structure [99, p. 51].

there will be a class of qualities γ one and only one of which attaches to any stretch on a straight line or on a point, such that matching in respect to this quality is what we mean by congruence.

The thesis that I have been maintaining is that measurement presupposes a perception of matching in quality. Accordingly in examining the meaning of any particular kind of measurement we have to ask, What is the quality that matches? [99, p. 57].

a yard measure is merely a device for making evident the spatial congruence of the [extended] events in which it is implicated [99, p. 58].

Let us now examine the several strands in Whitehead's argument in turn. We shall begin by inquiring whether his historical observation that the human race possessed a time metric prior to the enunciation of Newton's laws during the seventeenth century can serve to invalidate Poincare's contentions [64] that (1) time congruence in physics is conventional, (2) the definition of temporal congruence used in refined physical theory is given by Newton's laws, and (3) we have no direct intuition of the temporal congruence of nonadjacent time intervals, the belief in the existence of such an intuition resting on an illusion.

To see how unavailing Whitehead's historical argument is, consider first the spatial analogue of his reasoning. We saw in Section 3, part (iii) that although the demand that Newton's laws be true does uniquely define temporal congruence in the one-dimensional time continuum, it is not the case that the requirement of the applicability of Euclidean geometry to a table top similarly yields a unique definition of spatial congruence for that two-dimensional space. For the sake of constructing a spatial analogue to Whitehead's historical argument, however, let us assume that, contrary to fact, it were the case that the requirement of the Euclideanism of the table top did uniquely determine the customary definition of perfect rigidity. And now suppose that a philosopher were to say that the latter definition of spatial congruence, like all others, is conventional. What then would be the force of the following kind of Whiteheadian assertion: "Well before Hilbert rigorized Euclidean geometry and even much before Euclid less perfectly codified the geometrical relations between the bodies in our environment, men used not only their own limbs but also diverse kinds of solid bodies to certify spatial equality"? Ignoring now refinements required to allow for substance-specific distortions, it is clear that, under the assumed hypothetical conditions, we would be confronted with logically independent definitions of spatial equality issuing in the same congruence class of intervals. The concordance of these definitions would indeed be an impressive empirical fact, but it could not possibly refute the claim that the one congruence defined alike by all of them is conventional.

Precisely analogous considerations serve to invalidate Whitehead's historical argument regarding time congruence, if we discount Milne's hypothesis of the incompatibility of the congruences defined by "atomic" and "astronomical" clocks (cf. Section 4, part (iii)) and consider the agreement obtained after allowance for substance-specific idiosyncrasies between the congruences defined by a class of devices located in vanishing or stationary gravitational fields. A candle always made of the same material, of the same size, and having a wick of the same material and size burns very nearly the same number of inches each hour. Hence as early as during the reign of King Alfred (872–900), burning candles were used as rough time keepers by placing notches or marks at such a distance apart that a certain number of spaces would burn each hour [50, pp. 53–54]. Ignoring the relatively small variations of the rate of flow of water with the height of the water column in a vessel, the water clock or clepsydra served the ancient Chinese, Byzantines, Greeks, and Romans [67], as did the sand clock, keeping very roughly the same time as burning candles. Again, an essentially frictionless pendulum oscillating with con-
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stant amplitude at a point of given latitude on the earth defines the same
time metric as do “natural clocks,” i.e., quasi-closed periodic systems [5].
And, ignoring various refinements, similarly for the rotation of the earth,
the oscillations of crystals, the successive round trips of light over a fixed
distance in an inertial system, and the time based on the natural periods
of vibrating atoms or “atomic clocks” [93; 42; 2; 46].

Thus, unless Milne is right, we find a striking concordance between
the time congruence defined by Newton’s laws and the temporal equality
furnished by several kinds of definitions logically independent of that
Newtonian one. This agreement obtains as a matter of empirical fact (cf.
Section 4, part (i)) for which the GTR has sought to provide an explana-
tion through its conception of the metrical field, just as it has endeavor-
ted to account for the corresponding concordance in the coincidence behavior
of various kinds of solid rods [16, pp. 78–79]. No one, of course, would
wish to deny that of all the definitions of temporal congruence which
yield the same time metric as the Newtonian laws, some were used by
man well before these laws could be invoked to provide such a definition.
Moreover, Whitehead might well have pointed out that it was only be-
cause it was possible to measure time in one or another of these pre-New-
tonian ways that the discovery and statement of Newton’s laws became
possible. But what is the bearing of these genetic considerations and of
the (presumed) fact that the same time congruence is furnished alike by
each of the aforementioned logically independent definitions on the issue
before us? It seems quite clear that they cannot serve as grounds for
impugning the thesis that the equality obtaining among the time intervals
belonging to the one congruence class in question is conventional in the
Riemann-Poincaré sense articulated in this essay: this particular equality
is no less conventional in virtue of being defined by a plethora of physical
processes in addition to Newton’s laws than if it were defined merely by
one of these processes alone or by Newton’s laws alone.

Can this conclusion be invalidated by adducing such agreement as does
obtain under appropriate conditions between the metric of psychological
time and the physical criterion of time congruence under discussion? We
shall now see that the answer is decidedly in the negative.

Prior attention to the source of such concordance as does exist between
the psychological and physical time metrics will serve our endeavor to
determine whether the metric deliverances of psychological time furnish

any support for Whitehead’s espousal of an intrinsic metric of physical
time.55

It is well known that in the presence of strong emotional factors such
as anxiety, exhilaration, and boredom, the psychological time metric ex-
hibits great variability as compared to the Newtonian one of physics. But
there is much evidence that when such factors are not present, physiologi-
ical processes which are geared to the periodicities defining physical time
congruence impress a metric upon man’s psychological time and issue in
rhythmic behavior on the part of a vast variety of animals. There are two
main theories at present as to the source of the concordance between the
metrics of physical and psychobiological time. The older of these main-
tains that men and animals are equipped with an internal “biological
clock” not dependent for its successful operation on the conscious or un-
conscious reception of sensory cues from outside the organism. Instead
the success of the biological clock is held to depend only on the occur-
currence of metabolic processes whose rate is steady in the metric of physical
clock time [30, 39, 40]. As applied to humans, this hypothesis was sup-
ported by experiments of the following kind. People were asked to tap
on an electric switch at a rate which they judged to be a fixed number of
times per second. It was found over a relatively small range of body tem-
peratures that the temperature coefficient of counting was much the same
as the one characteristic of chemical reactions: a two or threefold increase
in rate for a 10° C rise in temperature [17]. The defenders of the concep-
tion that the biological clock is purely internal further adduce observa-
tions of the behavior of bees: both outdoors on the surface of the earth
and at the bottom of a mine, bees learned to visit at the correct time each
day a table on which a dish of syrup was placed daily for a short time at a
fixed hour. Having been found to be hungry for sugar all day long, neither
the assumption that the bees experience periodic hunger, nor the ap-

55 It will be noted that Whitehead does not rest his claim of the intrinsicality of the
temporal metric on his thesis of the atomicity of becoming. We therefore need not
deal here with the following of his contentions: (i) becoming or the transience of
“now” is a feature of the time of physics, the bifurcation of nature being philosophi-
cally illegitimate, and (ii) there is no continuity of becoming but only becoming of
continuity [101, p. 53]. But the reader is referred to F. S. C. Northrop’s rebuttal to
Whitehead’s attack on bifurcation [57], to my demonstration of the irrelevance of be-
coming to physical (as distinct from psychological) time [32, Sec. 4], to my critique
[37] of Whitehead’s use of the “Dichotomy” paradox of Zeno of Elea to prove that
time intervals are only potential and not actual continua, and to my essay “Whitehead’s
Philosophy of Science,” Philosophical Review, April 1962, for a defense of bifurcation.
pearance of the sun nor yet the periodicities of the cosmic ray intensity can explain the bees' success in time keeping. But dosing them with substances like thyroid extract and quinine, which affect the rate of chemical reactions in the body, was found to interfere with their ability to appear at the correct time.

More recently, however, doubt has been cast on the adequacy of the hypothesis of the purely internal clock. A series of experiments with fiddler crabs and other cold-blooded animals [6, 7] showed that these organisms hold rather precisely to a 24-hour coloration cycle (lightening-darkening rhythm) regardless of whether the temperature at which they are kept is 26 degrees, 16 degrees, or 6 degrees centigrade, although at temperatures near freezing, the color clock changes. It was therefore argued that if the rhythmic timing mechanism were indeed a biochemical one wholly inside the organism, then one would expect the rhythm to speed up with increasing temperature and to slow down with decreasing temperature. And the exponents of this interpretation maintain that since the period of the fiddler crab's rhythm remained 24 hours through a wide range of temperature, the animals must possess a means of measuring time which is independent of temperature. This, they contend, is "a phenomenon quite inexplicable by any currently known mechanism of physiology, or, in view of the long period-lengths, even of chemical reaction kinetics" [7, p. 159]. The extraordinary further immunity of certain rhythms of animals and plants to many powerful drugs and poisons which are known to slow down living processes greatly is cited as additional evidence to show that organisms have daily, lunar, and annual rhythms impressed upon them by external physical agencies, thus having access to outside information concerning the corresponding physical periodicities. The authors of this theory admit, however, that the daily and lunar-tidal rhythms of the animals studied do not depend upon any now known kind of external cues of the associated astronomical and geophysical cycles [7, pp. 153, 166]. And it is postulated [7, p. 168] that these physical cues are being received because living things are able to respond to additional kinds of stimuli at energy levels so low as to have been previously held to be utterly irrelevant to animal behavior. The assumption of such sensitivity of animals is thought to hold out hope for an explanation of animal navigation.

We have dwelled on the two current rival theories regarding the source of the ability of man (and of animals) to make successful estimates of duration introspectively in order to show that, on either theory, the metric of psychological time is tied causally to those physical cycles which serve to define time congruence in physics. Hence when we make the judgment that two intervals of physical time which are equal in the metric of standard clocks also appear congruent in the psychometry of mere sense awareness, this justifies only the following innocuous conclusion in regard to physical time: the two intervals in question are congruent by the physical criterion which had furnished the psychometric standard of temporal equality both genetically and epistemologically. How then can the metric deliverances of psychological time possibly show that the time of physics possesses an intrinsic metric, if, as we saw, no such conclusion was demonstrable on the basis of the cycles of physical clocks?

As for spatial congruence, what are we to say of Whitehead's argument [100, p. 56] that just as it is an objective datum of experience that two phenomenal color patches have the same color, i.e., are "color-congruent," so also we see that a given rod has the same length in different positions, thus making the latter congruence as objective a relation as the former? As Whitehead puts it: "It is at once evident that all these tests [of congruence by means of steel yard measures, etc., are] dependent on a direct intuition of permanence" [101, p. 501]. He would argue, for example, that in the accompanying diagram the horizontal segment AC could not be stipulated to be congruent to the vertical segment AB. For the deliverances of our visual intuition unequivocally show AC to be shorter than AB and AB to be congruent to AD, a fact also attested by the finding that a solid rod coinciding with AB to begin with and then rotated into the horizontal position would extend over AC and coincide with AD.

On this my first comment is to ask: What is the significance for the status of the metric of physical as distinct from visual space of these observational deliverances? And I answer that their significance is entirely consonant with the conventionalist view of physical congruence expounded above. The criterion for ocular congruence in our visual field was presumably furnished both genetically and epistemologically by ocular adaptation to the behavior of transported solids. For when pressed as to what it is about two congruent-looking intervals that enables them to sustain the relation of spatial equality, our answer will inevitably have to be this: the fact of their capacity to coincide successively with a trans-
ported solid rod. Hence when we make the judgment that two intervals of physical space with which transported solid rods coincide in succession also look congruent even when compared frontally purely by inspection, what this proves in regard to physical space is only that these intervals are congruent on the basis of the criterion of congruence which had furnished the basis for the ocular congruence to begin with, a criterion given by solid rods. But the visual deliverance of congruence does not constitute an ocular test of the “true” rigidity of solids under transport in the sense of establishing the factuality of the congruence defined by this class of bodies. Thus, it is a fact that in the diagram AD extends over (includes) AC, thus being longer. And it will be recalled that Riemann’s views on the status of measurement in a spatial continuum require that every definition of “congruent” be consistent with this kind of inclusional fact. How then can visual data possibly interdict our calling AC congruent to AB and then allowing for the de facto coincidence of the rotated rod with AB and AD by assigning to the rod in the horizontal position a length which is suitably greater than the one assigned to it in the vertical orientation?

It will be recalled (cf. Section 5, part (i)) that Russell had unsuccessfully sought to counter Poincaré’s position by answering the question “What is it that is measured?” on the basis of the affirmation of an intrinsic metric. Whitehead believes himself to have supplied the missing link in Russell’s answer by having added the deliverances of visual space and of psychological time. It remains for us to consider briefly the further reasons put forward by Whitehead in his endeavor to show the following: transported rods and the successive periods of clocks can be respectively held to be truly unaltered or congruent to themselves, thereby rendering testimony of an intrinsic metric, because an intuitively apprehended matching relation obtains between the visual and psychotemporal counterparts of the respective space and time intervals in question.

Invoking visual congruence, Whitehead claims [97, p. 121] that an immediate perceptual judgment tells us that whereas an elastic thread does not remain unaltered under transport, a yard measure does. And from this he draws three conclusions [97, p. 121]: (i) “immediate judgments of congruence are presupposed in measurement,” (ii) “the process of measurement is merely a procedure to extend the recognition of congruence to cases where these immediate judgments are not available,” and (iii) “we cannot define congruence by measurement.” The valid core of assertions (i) and (iii) is that measurement presupposes (requires) a congruence criterion in terms of which its results are formulated. But this does not, of course, suffice to show that the congruence thus presupposed is nonconventional. Neither can the latter conclusion be established by Whitehead’s contention that we apprehend a matching relation among those intervals which are congruent according to the customary standards of rigidity (or isochronism). For the matching among intervals which are congruent relatively to a rigid rod is only with respect to such metrical nonintrinsic properties as the coincidence of each of them with that transported rod, or as the round trip times required by light to traverse them in both directions in an inertial system. To this, Whitehead retorts with the declaration “There is a modern doctrine that ‘congruence’ means the possibility of coincidence. . . . although ‘coincidence’ is used as a test of congruence, it is not the meaning of congruence” [101, p. 501]. The issue raised by Whitehead here is, of course, one of uniqueness and intrinsicness of equality among intervals. We are therefore not concerned with the separate point that no one physical criterion can exhaustively specify “the meaning” of the open cluster concept of congruence as applied to any particular congruence class of intervals. Accordingly, we ignore here refinements that would allow for the open cluster character of physical congruence. And we point out that if there were a meaning in the ascription of an intrinsic metric to space, then it would be quite correct to regard coincidence as only a test of congruence in Whitehead’s sense of ascertaining the existence of intrinsic equality. For in that case one would be able to speak of two separated intervals as matching spatially in the sense of containing the same intrinsic amount of space. But it is the existence of an intrinsic metric which is first at issue. And the position taken by Whitehead on that issue cannot be justified without begging the question by simply asserting that coincidence is only the test but not the meaning of congruence. Hence, his argument from visual congruence having failed, as we saw, Whitehead has not succeeded in refuting the conception that (1) the matching lies wholly in the objective coincidences of each of two (or more) intervals with the same transported rod and (2) the self-congruence of the rod under transport is a matter of convention.

It is significant, however, that there are passages in Whitehead where

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*Cf. also [100, pp. 54–55].*
he comes close to the admission that the preeminent role of certain classes of physical objects as our standards of rigidity and isochronism is not tantamount to their making evident the intrinsic equality of certain spatial and temporal intervals. Thus speaking of the space-time continuum, he says: "This extensive continuum is one relational complex in which all potential objectifications find their niche. It underlies the whole world, past, present, and future. Considered in its full generality, apart from the additional conditions proper only to the cosmic epoch of electrons, protons, molecules, and star-systems, the properties of this continuum are very few and do not include the relationships of metrical geometry" [101, p. 103]. And he goes on to note that there are competing systems of measurement giving rise to alternative families of straight lines and correspondingly alternative systems of metrical geometry of which no one system is more fundamental than any other [101, p. 149]. It is in our present cosmic epoch of electrons, protons, molecules, and star systems that "more special defining characteristics obtain" and that "the ambiguity as to the relative importance of competing definitions of congruence" is resolved in favor of "one congruence definition" [101, p. 149]. Thus Whitehead maintains that among competing congruence definitions, "That definition which enters importantly into the internal constitutions of the dominating . . . entities is the important definition for the cosmic epoch in question" [101, p. 506]. This important concession thus very much narrows the gap between Whitehead's view and the Riemann-Poincaré conception defended in this essay, viz., that once a congruence definition has been given conventionally by means of the customary rigid body (or otherwise), then, assuming the usual physical interpretation of the remainder of the geometrical vocabulary, the question as to which metric geometry is true of physical space is one of objective physical fact. That the gap between the two views is narrowed by Whitehead's concession here becomes clear upon reading the following statement by him in the light of that concession. Speaking of Sophus Lie's treatment of congruence classes and their associated metric geometries in terms of groups of transformations between points (cf. part (i) of Section 5), Whitehead cites Poincaré and says:

The above results, in respect to congruence and metrical geometry, considered in relation to existent space, have led to the doctrine that it is intrinsically unmeaning to ask which system of metrical geometry is true of the physical world. Any one of these systems can be applied, and in an indefinite number of ways. The only question before us is one of convenience in respect to simplicity of statement of the physical laws. This point of view seems to neglect the consideration that science is to be relevant to the definite perceiving minds of men; and that (neglecting the ambiguity introduced by the invariable slight inexactness of observation which is not relevant to this special doctrine) we have, in fact, presented to our senses a definite set of transformations forming a congruence-group, resulting in a set of measure relations which are in no respect arbitrary. Accordingly our scientific laws are to be stated relevantly to that particular congruence-group. Thus the investigation of the type (elliptic, hyperbolic or parabolic) of this special congruence-group is a perfectly definite problem, to be decided by experiment [98, p. 265].


Though Whitehead's argument that observed matching relations attest the existence of an intrinsic metric is faulty, as we saw, that argument can no more be dismissed on the basis of the model-theoretic trivialization of the congruence issue than Russell's argument against Poincaré (cf. Section 5, part (i)). In fact, one should have supposed that those who maintain with Eddington that GC is a subthesis of TSC (cf. Section 2, part (i)) would have suspected that their critique had missed the point. For what should have given them pause is that Russell, Whitehead, and, for that matter, Poincaré were clearly aware of the place of geometry in the theory of models of abstract calculi and yet carried on their philosophical polemic regarding the status of congruence. According to the triviality thesis, the stake in their controversy was no more than the pathetic one that Russell and Whitehead were advocating the customary linguistic usage of the term "congruent" (for line segments) while Poincaré was maintaining that we need not be bound by the customary usage but are at liberty to introduce bizarre ones as well. Thus, commenting on Poincaré's statement that we can always avail ourselves of alternative metrizability to give a Euclidean interpretation of any results of stellar parallax measurements ([63, p. 81]; cf. Section 6 below), Eddington writes:

Poincaré's brilliant exposition is a great help in understanding the problem now confronting us. He brings out the interdependence between geometrical laws and physical laws, which we have to bear in mind continu-