Underlying a large number of proposals in cognitive theory is the conviction that mental activity should be seen as a series of discrete computational processes on sets of elements. It is felt, in turn, that formal programs or systems describing such computations provide us with the best (and perhaps the only possible) psychological theories of human cognition. Of course, once one settles on such a model of cognition, other things follow. In particular, there is the strong push to go from truths about the limits and capacities of certain formal systems and how they function to claims about the limit of the mind and how the mind itself must function. By focussing on some arguments about the significance of infinite sets and unbounded competences, I hope to show some of the problems involved in making this move from formal theory to psychological reality.

Although it has seemed obvious that human capacities are physically limited and finite, it is often observed that people can acquire certain competences that are unbounded or unlimited. This discrepancy between limited capacities and unbound competences is thought to have important psychological implications, particularly with regard to learning. In most cases, however, the resulting claims made about the mind are not themselves supported by empirical evidence; rather they are usually thought of as self-evident truths to be freely used as premises in support of some further position or theoretical point. In this chapter I intend to examine several arguments about language that depend on claims concerning infinity.

Note: I wish to thank M. Atherton, S. Morgenbesser, and D. Rosenthal for discussing some of these matters with me, and C. W. Savage for much helpful editorial advice. This work was supported by a Faculty research Award from the City University of New York.
I shall first present a sketch of an argument (A) and then examine (E) the psychological significance of the conclusions reached. I shall make no effort to attribute these arguments to individual theorists, nor shall I attempt to criticize specifically any theorist's pet versions of these arguments. For what I mainly wish to show is that infinitary arguments of this sort go through only if we make some additional assumptions about the nature and limits of mind or mental processes. And further, I wish to show that the assumptions needed are not at all obvious truths about cognitive capacity in general, but require supplementary evidence or argument to justify their use in individual cases. In particular, I think the additional assumptions necessary to make the infinitary arguments sound in the case of natural language are not readily extendible when we consider the full range of human symbolic competences, and even less so when we try to encompass cognitive skills in general.

Again, what interests me in each of the following examples is not the formal claim about the infiniteness of a set of items, but the psychological claim that is supposed to follow from it. For it is in shifting from formal fact to psychological conclusion that I believe unwarranted assumptions about the nature of mental processes are too readily accepted. Finally, it should be noted that I shall be using the terms 'infinite,' 'unlimited,' and 'unbounded' somewhat interchangeably throughout, not in their precise technical senses, but rather to describe sets of items for which there is no principled limit or cut-off point on the number of members they contain. I do this both for stylistic reasons and because, as I hope to show, the sense of infinity needed for arguments of this sort to proceed is itself part of the problem.

A Uniqueness Claim

(A) Natural languages are infinite. Linguistic competence is unbounded in the sense that the set of sentences a speaker of a natural language can produce and recognize as part of the language is theoretically infinite. This infinite nature of language makes linguistic competence qualitatively different from most other competences and thus shows the need for special learning capacities or processes not like those that might account for other skills.
(E) Whatever a correct account of language acquisition may turn out to be, there is nothing in the unboundedness of linguistic competence that serves to distinguish it from almost any other skill or competence we have. Our abilities to play chess, to identify baroque music, to add and subtract, to recognize and produce tables, chairs, triangles, doorknobs, the letter $A$, or, for that matter, almost anything else, are likewise unbounded skills. There is no fixed limit, for example, to the number of chairs we can recognize, and given some design skills, no fixed limit on the number of different kinds of chairs we can produce. This competence cannot reasonably be seen to involve the mastery of some list of chairs whose membership roll is consulted in each act of recognition or production. Not all chairs (or even kinds of chairs) to be dealt with will have been actually encountered in learning the concept ‘chair,’ and the creative designer can produce chairs vastly unlike those ever previously experienced. There is no reason to suppose that each new chair recognized or produced will be a mere copy or physically “similar” to earlier ones. Chairs may vary in color and size, in shape, style, and material, in beauty and comfort, and in whether or not they can function as seats. Chairs cannot be ordered or differentiated along some one or few linear stimulus dimensions. And to claim that identifying chairs is a case of mere pattern recognition, whereas determining which strings of words are grammatical is not, will remain a vacuous retreat, unless some cogent arguments can be given as to why the latter too is not a case of pattern recognition. Chairs are similar to one another in their chairness, and grammatical sentences are similar to one another in their grammaticality. Moreover, normal use of this chair competence would appear to be innovative and free from the control of detectable stimuli in ways not easily distinguishable from those assumed for language. The notions ‘innovative’ and ‘stimulus-free’ are not very clear, and a detailed analysis of them would sidetrack my main concern (but see Atherton, 1970). Loosely, my point is that we can design chairs to fit or to be appropriate to new situations and conditions, and that our designing activity does not in any simple way seem to be under the control of stimuli. All of these points could be made equally well about almost any other interesting competence we have mastered.
Now I am not here arguing that there are no important distinctions to be drawn between our various nonlinguistic skills and linguistic competence; nor am I maintaining that any creature capable of learning one must be capable of learning some or all of the others. My point is that if we are looking for characteristic differences among competences that will have psychological import, emphasis on the fact that the set of grammatical strings is infinite is misguided. If linguistic competence is to be distinguished from all these other skills, some feature other than unbounded recognition and productivity will be required. One obvious feature that would seem to separate linguistic competence from such skills as chair designing, chess playing, bicycle-riding, etc. is that our linguistic skill is essentially a symbolic skill—its products can stand for, denote, refer, or describe. To the extent that we focus our attention solely on our ability to pick out syntactically grammatical strings, there would seem to be little reason to distinguish this skill from other pattern recognition competences, for, by itself, this skill is hardly a symbolic competence. But, of course, to have learned a natural language, we must know how sentences relate to the world. It is in our having acquired an associated skill of interpretation that our manipulation of marks and utterances becomes a symbolic skill. So, if it is the symbolic nature of linguistic skill that is psychologically distinctive and important, then it is semantic competence that looms large.

Clearly any adequate theory of verbal learning must be able to account not only for our ability to recognize grammaticality, but for the way we learn the referential or descriptive force of sentences. And it might be thought that here considerations of infinity would lend uniqueness to language competence. For not only can we recognize the grammaticality of a theoretically infinite set of sentences, we also know how to interpret them, we know what conditions would make them true. We can produce and understand sentences never heard before, describing situations never before encountered. So it may be thought that the need to account for an unbounded semantics is what particularizes theories of natural language acquisition.

But again little argument is needed to show that if we are to establish the uniqueness of natural language competence, something other than appeal to the infinite is required. For even if we limit our consideration to symbolic skills, to skills that have an essential
semantic or referential component, the unboundedness of natural language competence will not be distinctive. Most other symbol systems that we use every day (e.g., maps, models, pictures, diagrams, music notation) are likewise unbounded. Mastering any one of these systems enables us to recognize and understand the descriptive force of an unlimited set of new symbols and to use these symbols creatively to represent or describe an unlimited set of novel situations. Furthermore, unlike mastering some systems of animal communication, mastering these symbol systems requires more than being able to correlate a single stimulus dimension with some single property of the symbol scheme. And in general there will be no regular relationship between producing or understanding a given symbol in these systems and any immediately ensuing behavior pattern or emotional state. Nor do I think it can be argued, as it is sometimes said of artificial languages, that these systems all presuppose natural language or are merely notational variants of or parasitical on natural language.

So although the need to master the theoretically infinite semantic force of natural language may be of importance in distinguishing linguistic competence from various other skills, it will not separate out this competence from most of our symbolic skills. From the standpoint of learning theory, further differentiation would depend upon exposing distinctions between natural language competence and our other symbolic skills, and then providing empirical evidence that these distinctions have a critical effect on learning. Thus, for example, Katz (1972, pp. 22 ff), arguing along somewhat narrower lines than I, claims that the infinity of sentences will not set natural language apart from various other languages. Rather, he thinks natural languages are distinguished from other languages by their superior expressive power. I have qualms with aspects of this latter claim, which I am not going to pursue here. If Katz were correct, however, one might then want to show how the supposedly superior expressive power of natural language requires special learning capacities or processes.

From Infinity to Rules

(A) The infiniteness of natural language entails that anyone possessing the competence to understand or speak such a language must employ a set of rules.
Rather than the term "employ." If the claim that
rules must be employed is merely a way of saying that linguistic
competence is an open-ended skill and that performance is not based
on consulting an exhaustive list, then the claim is undoubtedly true.
As we have suggested, hardly any interesting competence can be seen
to involve the learning or memorization of a list that is consulted
in each case. If, however, the argument is that the infiniteness of lan-
guage entails the use of rules, that the speaker employs rules in rec-
ognition, understanding, and producing language, then it seems that
without some additional empirical premises the inference is unsound.

From a logical standpoint the argument from infinity to rules
breaks down; it leads to a regress. If rules are construed as internal-
ized instructions, formulas, or principles that specify membership
conditions or describe the members of a given infinite set, use of
such rules would require the ability to apply the concepts or criteria
employed in the rule specification. But the skills needed to apply
the rule to a theoretically unlimited number of cases would them-
selves be unbounded and thus require the use of additional rules,
and so on ad infinitum. For example, suppose the skill under con-
sideration is that of recognizing $P$s, and suppose $x$ is a $P$ if and only
if $x$ has properties $Q_1$, $Q_2$, ..., $Q_n$. Determining the presence of $Q_1$,
$Q_2$, ..., $Q_n$ provides a rule for picking out $P$s. However, if the set of
$P$s is unlimited, the ability to apply $Q_1$, $Q_2$, ..., $Q_n$ must also be
unlimited, and this skill would require the use of yet another set
of rules, those that permit us to pick out cases of $Q_1$, $Q_2$, ..., $Q_n$.

So if we are to avoid a regress, we must allow that there are at least
some unbounded competences which themselves are not to be ac-
counted for in terms of employing rules. Thus in our example it
might be claimed that the abilities to recognize $Q_1$, $Q_2$, ..., $Q_n$, al-
though unbounded skills, cannot be further reduced. But then it
must be allowed that some other type or types of psychological
processes can also account for unbounded competences. And in any
particular case, determining which model will account for which
competence remains an empirical matter. (Morgenbesser, 1969,
Nagel, 1969, and Malcolm, 1971 have interesting things to say about
the problems under consideration here and in what follows.)

Now, for many of our skills, I would argue that the rule-following
model is unlikely to prove useful as a psychological explanation.
What, for example, is the significance of the claim that we master rules when we learn to recognize and produce chairs, or the letter “A”? Clearly most of us could not state a definition or rule that would enable us to pick out all chairs, or all “A”s. And even if we were to do so, there would be no good reason to assume that we had been unconsciously consulting the rule all along, or that it had played a role in bringing about our performance. That such definitions might pick out the class of objects that we call “chairs,” or “A”s, does not by itself show that these definitions play any role in our processes of recognition or production. Indeed, the odds are that a satisfactory definition of “chair” or “letter A” will involve concepts more complex and difficult to apply than the predicates “chair” and “letter A” themselves. And, as our previous argument was meant to show, we cannot, on pain of regress, assume that each of our unbounded recognition and production skills requires the literal use of rules. Some of our skills must be irreducible, i.e., not explainable on a rule-following model.

But our chair and letter A competences seem to me to be as good candidates for unreduced status as any other. The mere fact that they are learned skills does not entail a rule mastery account of our present competence. The push to view all our learned cognitive competences as literally composed of or definable in combinatorial terms from some fixed set of elementary concepts results from assuming a priori that a reductionist computational model must be correct. On the other hand, I think it more reasonable to assume that many of our skills function as our unreduced skills must be thought to do. Hence these developed skills will not be adequately modeled by the sorts of reductive programs the computational approach usually promotes. Arguing for this latter position, though, would be a lengthy matter. What I have attempted to establish here is that for any given competence, empirical evidence is required to show that its use is mediated by the sort of processing a literal construal of the rule model would seem to demand.

It is true, perhaps, that linguistic competence may on the face of it appear to be a more promising realm for employing a rule model of processing. However, I do not believe we have at present adequate evidence or grounds for assuming it to be the only or most appropriate model for understanding these matters. This, of course,
is not to imply that we cannot talk about the underlying neurological and/or psychological processes that accompany our linguistic accomplishments. My question is whether the set of formal rules that a grammarian or logician may propose provides such an account, and if so, what sort of account it provides. Are these rules merely descriptions of certain regularities in the speaker’s competence, in the way that one or another axiomatization of logic may be seen as a representation of the ideal logician’s reasoning competence; or is there some stronger sense in which the speaker can be said actually to employ or follow the rules? If the connection between rules and competence is thought of as similar to the logic case, then any direct application of the rule-model as a psychological account would seem an unpromising approach to learning theory. If a more central explanatory role is to be assigned to the rules of grammar, it would seem that some process, state, or internal phenomena must be seen as embodying, representing, or reflecting the particular rules. Just what such an embodiment of rules would reasonably be like and how it would play a role in verbal behavior I find unclear. I would claim, however, that recent attempts to explicate rule-mastery in terms of tacit or implicit knowledge do not help matters much. For the issue here at least is not whether linguistic competence is best described as having knowledge, tacit or otherwise. Rather, the main question is whether anything like a formal grammar plays a role in our processes of production and understanding, whether some mechanism involved in verbal performance actually goes about assigning sentences the full analysis a grammar does.

But it is not possible, nor would it be to the point, to rehash here the many problems involved in the idea of tacit knowledge and linguistic competence. More on this matter can be found in Harman (1967), Hook (1969), and elsewhere. For our primary task has been to show that claims about the routes of language-learning and performance do not follow by themselves, once it is noted that natural languages are formally infinite. If a rule-model of competence can be made clear and plausible, still justification for applying such a model to linguistic competence would require more than an appeal to the infinite. It would be necessary to demonstrate some particular features of linguistic skill that recommend a rule-model. And I should think that whatever these features turn out to be, they would
continue to suggest such a rule-based psychological account whether the sentences of some natural language were finite or infinite in number.

The Assumption of a Finite Base

(A) The number of semantically primitive expressions of a language must be finite; otherwise the language would be unlearnable.

(E) The rationale for this claim seems obvious. The words "cow," "ink," and "shoe" each denote kinds of objects, and what each denotes is a matter of convention. There is no a priori reason why the word "cow" is used to denote cows rather than shoes and why "shoe" is used to denote shoes rather than ink. So it would seem that if the inventory of terms were infinite, we could not possibly learn the denotation of them all. Thus it is assumed that, for a language to be learnable, it must contain only a finite number of primitives.

Now the major problem in evaluating this claim is that the notion of a "semantic primitive" in a natural language is not a very clear one. A natural language, as opposed to an artificial language, is not given to us with a list prescribing its primitive vocabulary and a set of official definitions sanctioned in the system. Terms are usually characterized as primitive only relative to a particular formalization of a language, since expressions primitive in one formalization may be defined in another and vice versa. But even if a particular set of terms is not to be singled out as representing the primitives, when the system we are dealing with is not an artificially constructed one, perhaps the force of the learnability claim can still be made with a more relative notion of primitives. Suppose, as a first approximation, we allow that a term is primitive relative to the other terms in a set if it cannot be defined or its semantics specified using only these other terms. We can then interpret the psychological assumption as saying that, if the terms or semantic units of a system are infinite, the system will be unlearnable, unless it is possible to specify the semantics of the set using only some finite subset of itself.

But on this formulation of the learnability claim, is there any reason to believe it is a true psychological principle? After all, we do master certain unbounded symbol systems in which it seems unlikely that we can finitely specify the semantic significance of all members in terms of the content of some of the others. Consider
such symbol systems as gestures (e.g., mime), models, or pictures. In these systems, each different gesture, model, or picture has its semantic function, and each seems to be primitive with respect to the other symbols in the system. In such symbol systems, I do not see the grounds for claiming that the semantic units of the system are reducible to a finite set. But surely these systems are learnable, and there is no reason to suppose that learning them must proceed via reduction to or specification of their semantics in some other system like natural language. What I am arguing, then, is that given practice in reading or interpreting gestures, models, and pictures, we can project from the finite teaching instances in a way that allows us to comprehend the semantic significance of a potentially infinite set of distinct symbols.

Now one major difference between natural languages and gestures, models, and pictures is that natural languages are syntactically notational whereas these other schemes are syntactically dense—they are nonnotational or analogue schemes. In the case of natural language we can isolate an alphabet and vocabulary of repeatable characters, and these characters are separable into disjoint, differentiated classes. (Goodman, 1968, develops these distinctions more precisely and in detail.) In the analogue cases division of the schemes into component characters of this kind is not possible. Perhaps it will be maintained that only in systems of this latter analogue sort can semantic learning proceed unbounded. Although this distinction among symbolic schemes marks a significant difference, I do not think it is one that can rescue this version of the learnability claim. Take, for example, a subsystem of standard Western music notation—a system having a vocabulary of only whole notes, each denoting a class of sounds, and containing no sharps or flats. Such a system is disjoint, notational, and not analogue. Within such a system there will be no way to define or specify the semantical force of a note in terms of other notes in the vocabulary, and if the system provides for higher and lower notes without limit, the vocabulary of notes will be theoretically infinite. And yet his system seems clearly learnable by inductive means. By providing someone with a finite set of teaching instances (a written note accompanied by a sound it denotes) the person will eventually be able to project to new cases. Shown a written note not among the teaching instances, the learner will understand what
it says, and there will be no fixed limit on the number of new notes capable of being so handled.

I am not talking here about new combinations of previously learned notes, but of understanding new, hitherto unseen individual notes. There will, of course, be limits to how many new notes can actually be distinguished and applied, but such limitations would seem to be more a matter of bounds on discriminatory power rather than a limit on our ability to master semantic systems. According to our present interpretation of primitiveness, then, each of these notes will be construed as a semantic primitive, and their number is theoretically infinite. The obvious objection, that each is not really a primitive, will be taken up shortly.

But are there any sets of expressions in natural language that are like gestures, models, pictures, and music notation in that they are unlimited and learnable on the basis of selected instances? Right off it would seem that there are. Consider the indicator or token reflexive words "I," "here," "that," "you," etc. Tokens of each of these words have different denotations, and there is no fixed number of such tokens we can comprehend. Given some instances of these words, we seem to learn how to handle new cases. Mother, father, and sister use "I," and we are able to project from these instances the correct denotation of our aunt's "I." Of course there is the difference with indicator terms that each token, although possibly differing in denotation, is of the same type or is an instance of the same character in the language. However, there are examples of infinite sets of terms differing in type in which it seems possible that inductive learning could occur. Consider a vocabulary in which the numeral "1" is used to denote one-inch long objects, "2" to denote two-inch long objects, etc. Take someone with no mathematical vocabulary and teach this measurement system by inductive means, i.e., pair numerals with objects of the appropriate length. After suitable practice in associating numerals with objects, it is likely that the person will learn how to go on, how to apply new numerals correctly, and there would seem to be no fixed upper limit on the learner's ability to continue. There may be cases, then, in natural language similar to gestures, models, pictures, and music, in which we are able to work out how to handle an unlimited vocabulary from a finite number of teaching instances.
Of course the objection will be raised that in the case of "I" and this measurement system (as well as the music system and perhaps even the analogue systems mentioned), although the vocabulary is infinite, we really are not dealing with sets of infinite primitives. After all, it might prove possible to define or reduce the semantic import of the various occurrences of "I" to only one expression, such as "the present speaker," and we can finitely specify the semantics for the size numerals using only a few mathematical terms like "plus," "times," "power of," etc. So in these cases we are really dealing with finite sets of primitives.

The problem with this reply is that it involves a slight but psychologically important shift in the notion of a primitive. For notice that, although others may have resources in their vocabulary enabling them to specify finitely the semantics for these infinite sets, there is no reason to suppose that anyone who masters these sets also has the additional resources. Surely a child could learn to understand tokens of "I" before acquiring the expression "the present speaker." Similarly, although the set of numeral terms may not be primitive with respect to the mathematician's resources, there is no reason to assume such richness in every learner's vocabulary. And even if the learner does have this additional vocabulary, it is not at all likely that he or she will be able to specify the semantics for their infinite vocabulary with this finite base. If we or the child as language user cannot perform such definitional reduction or specification, however why isn't it the case that we have mastered an infinite set of primitives? From a psychological point of view, from considerations about how and what we can learn, the fact that someone else happens to be able to specify the semantics for our infinite vocabulary by finite means should not make any difference.

The reason such an external specification or reduction has been thought relevant to the issue of learning, I think, depends on the further assumption that something like this definitional reduction must psychologically underlie our semantic skill, or else we could not master an infinite vocabulary. It is assumed that in order for us to be able to extend our vocabulary in an unbounded way, we must have mastered a semantic rule. Somehow the reducing definition scheme marks more than a formalization of our semantics; it also plays a role in the underlying psychological processes, it serves
as a rule we "follow." Although we cannot consciously perform the definitional reduction, this is what our mind or nervous system "has done." But what evidence or proof do we have that semantic learning can be accomplished only by means of our minds defining or specifying the semantic force of new expressions in terms of some more basic or previously available vocabulary? As was suggested in the previous section of this paper, I am not at all maintaining that nothing can be said in the way of explaining our unbounded competences. The problem here is to determine whether or in what way any particular definition or formal specification by something that can reasonably be called a "semantic rule" supplies such an account.

In any case, it should be clear that, unless we put some restrictions on what is to count as a semantic rule or as mastering a semantic rule, it is hard to see how to begin to establish the claim that only finitely based languages are learnable, or, on the other hand, to know what it would even mean to acquire an infinite set of primitives. For example, if merely mastering an infinite vocabulary is taken as synonymous with having a semantic rule, then the limitations claim would be vacuous. We could not learn an infinite set of semantic primitives, for learning the set would mean the same as mastering a semantic rule, and this would ensure that the number of primitives is finite. Similarly, consider again the case with the indicator terms. We could perhaps finitely specify the denotations of each of these, if we were to allow certain semantic or pragmatic terms to occur in our rules. We might specify that tokens of "here" are true of just the areas the speaker is indicating, or that tokens of "you" denote the person or persons addressed by the speaker. But if we are willing to allow semantic and pragmatic notions like "indicate" or "address" into our rules, it may not even be possible for there to be a vocabulary containing an infinity of primitives. We might be able to specify the semantics for all predicates or singular terms of the lexicon with some sort of single rule, saying that for arbitrary \( P \), \( P \) refers to what the speaker indicates or denotes by using \( P \) in the sentence. But surely this would not prove anything about the number of primitives in the speakers language. (In a paper that just came to my attention, Cummins, 1975, addresses himself to aspects of these issues.)
And more importantly, returning to some of the nonlinguistic symbol systems we mentioned earlier, e.g., mime, models, and pictures, what sort of rules are we to allow in analyzing the semantic function of these symbols? Here it is at least doubtful that a componential semantic analysis of the sort usually offered for natural languages—analyses that start with an initial stock of words or morphemes and build up to sentences—can get off the ground, yet we can master these systems. But if we are to maintain the learning limitations claim in full generality, i.e., as a limit on the human ability to master symbolic systems, it would be necessary to show how these competences too are reducible to a finite number of semantic rules or primitives of the specified kind.

Obviously, if someone wishes to formalize finitely the semantics of a natural language or other symbol system, reducing the needed primitives to a finite stock will be in order. But such reduction will then be forced on semantic analysis by formal features of the type of analysis proposed and not by psychological demands. No claim about the impossibility of learning a language follows by itself from the fact that the language contains an infinite number of primitives on some particular analysis. Without additional assumptions, questions of formal analysis and questions of psychology remain separate issues. In order to support the claim that a language is unlearnable unless it turns out to be finitely based on a particular kind of analysis, it would be necessary to establish a connection between the analysis proposed and learning processes. It would be necessary to show how the proposed specification of the semantics of a given vocabulary is relevant to issues of acquisition. Only then might it be possible to begin to argue that unless the vocabulary can be finitely specified in that way, it could not be learned. As it is, I know of no argument yet proposed that goes to establish and give psychological content to these sorts of claims.

Nevertheless, at least in the case of some symbol systems like natural language, I think there is a point lying behind the assumed limit on primitives. It is just that the usual reasoning from infinity has matters backwards. The important difference between the set of words “shoe,” “ink,” “cow,” etc., and the other systems considered is that the items in this set are arbitrary relative to one another. They are arbitrary in the sense that learning one gives no substantial pur-
chase or bias toward learning the denotation of other members of the set. Learning the use of "shoe" and "ink" does not enable one to understand "cow," and vice versa. On the other hand, the members of the music vocabulary, measurement systems, indicator terms, and various of the analogue systems cited are not arbitrary in this way. Learning some members of the system does significantly guide and shape our understanding of other members. (I have discussed this issue in more detail in Schwartz, 1975.)

Arbitrariness of this sort, though, is essentially a question of learnability and not definability. Whether a set of items is to be considered arbitrary depends on whether or not the items provide a basis for learning one another. As I have argued, the connection between learning and any particular type of semantic specification will, from a psychological standpoint, always be derivative. We shall not be able to determine that a set of symbolic items is inductively unlearnable by appeal to the impossibility of a particular type of formal reduction unless it is first established that such a definitional specification marks the route by which learning proceeds (or that other routes exist only when such specification is possible). As well as I can interpret it, the original claim that we can learn only a finite number of primitives, especially if it is to be construed as a claim about limits on human symbolic capacity in general, borders on the tautological. For the only sense I have been able to make of primitiveness has been in terms of learnability. But given any set of items, finite or infinite, semantic or other, if it turns out that learning the use of some does not provide sufficient experience for learning the rest, we shall not be able to learn the rest. The claim that we can learn only systems containing a finite number of primitives would then amount to the claim that what we find out we cannot learn under a given set of conditions, we cannot learn under those conditions.

If all this is correct then it also follows that we shall not be able to force any particular kind of semantic analysis on a language merely by citing the formal infiniteness of a linguistic class. Only by requiring that a semantic theory represent some additional specific psychological facts can we derive psychological support for assigning semantic structure. From this point of view, the significant point underlying the assumed limit on primitives may, I think, be brought to light. The claim cannot be interestingly construed as an established
constraint on human symbolic capacities. Rather it might best be seen as suggesting a psychological criterion of adequacy for semantic theory. The criterion it suggests is that formal semantics should in some way indicate certain learning relationships among semantic items. Briefly, a semantic analysis should reflect such facts as that learning items of sort $A$ enables us to understand items of sort $B$. Justification then for assigning structure will depend on judgments about whether specific learning relationships hold among items and not on whether or how the items can be defined or reduced to one another.

Tokens of a given indicator term will, for example, be construed as instances of one lexical item because learning the reference of some tokens enables us to understand new tokens of that type. Whether the semantics for the indicator can or cannot be specified in one finite fell swoop without appeal to other semantic or pragmatic notions is not to the point. Similarly, on this account, semantic structure will be assigned to the phrases "decoy duck," "centaur picture," and possibly "ran quickly," not because the semantics of the composite phrase can readily be defined in terms of the denotations of its components, but because there is an interplay between the learning habits associated with other tokens of these components, occurring in other contexts and different composites, which serve to guide our construal of the composite expression, and vice versa. The vice versa is significant here. For notice I am not claiming any necessary inferential relationship between composites and their parts, nor that in understanding the composite we actually project its meaning from that assigned its parts in isolation, nor that learning the parts must come first. We may come to learn the parts by being taught the compounds, e.g., when we teach a child the meaning of "tiger" by teaching the child to pick out which things are correctly called "tiger-pictures." Likewise, by appealing to learning relationships, we would justify treating a term and a metaphorical use of the term as one lexical item, rather than as an ambiguous term with two lexical entries, etc. To claim, though, that such learning relationships exist between tokens is not to claim that they have the same meaning or denotation or identical role in their home sentences. To assume that some fixed common property must run through or lie behind all cases of semantic learning is to accept a priori a reductionist view of learning that I have been cautioning against.
Obviously these last suggestions are only a sketch of an approach; much more needs to be said before its implications can be drawn out. Also, as should be clear by now, extending the notions of semantic unit and semantic primitive to other kinds of symbol systems will involve serious complications. I mention this proposal here only because I think something like it, rather than any informative thesis about human symbolic capacities, lies at the heart of the original claim that we can learn only finitely based languages.

Concluding Remarks

The purpose of this paper has been to explore several psychological claims concerning the infinite. These infinitary arguments are initially very appealing, since they appear to offer us insight into the learning process merely by looking at the logic of the situation. The theses are seen as a priori or transcendental principles of human behavior and mental activity that can be established by formal analysis alone, without the need for empirical support or argument. But this is surely a mistake. What can be established by any such formal analysis is that, if humans are systems of a specific sort, if they accept and process data in a certain way, then a particular limitation will exist. To show that a limitation actually exists, though, we must first show that humans are systems of that specific type, and that in a given case the material presented is processed in the specific way. In order to make the formal argument applicable or psychologically relevant, further assumptions must be added about the system at hand, which can only be justified by studying our actual competences and the ways we go about acquiring them. In the case of claims based on infinity, this is particularly striking. For the relevant notion of infinity will always be relative to some system of classifying, parsing, or individuating. The world comes to us neither as one nor as many. So we can establish infinity arguments intelligibly only after we have established something about how the given set is actually organized and processed, and only after we have understood what means are available to the organism for handling the material. Once this psychological study of the actual perceptual and learning processes is completed, moreover, it would seem that an appeal to the infinite will add relatively little. The important psychological insights will have been gained in determining what kind of a system the organism really is.
I have attempted to focus on this methodological issue concerning the relationship between formal theory and models of mind by examining several claims about learning natural language. I have argued that in each case a simple appeal to the infiniteness of some specified set is not sufficient to establish significant and interesting psychological principles. I have not sought to reject outright or to refute these various claims; for although it should be clear that I find these assumptions—as sometimes propounded—suspicious, I think there are psychologically interesting points lurking behind them. In support of this feeling, in some areas, I have even offered tentative suggestions as to what I think would be a fruitful approach to the problems. My hope is that by being more prudent in our reliance on the infinite, we can progress toward a better account of the underlying psychological issues.

References